Coordinate Geometry Study Summary

Distance Between Two Points

When we are asked to find the distance between two points on the number plane, we use the following formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Point 1 (1) Point 2 (2)

Find the distance between (-3,5) and (1,2).

$x_1 y_1$ $x_2 y_2$

x of point 1, y of point 1, x of point 2, y of point 2

$$\text{Distance} = \sqrt{(1 - -3)^2 + (2 - 5)^2}$$
$$\text{Distance} = \sqrt{(4)^2 + (-3)^2}$$
$$\text{Distance} = \sqrt{16 + 9}$$
$$\text{Distance} = \sqrt{25}$$
$$\text{Distance} = 5 \text{ units}$$

Point 1 (1) Point 2 (2)

Find the distance between (-3,5) and (1,2).

$x_1 y_1$ $x_2 y_2$

Variations

Find the exact distance between (4,-2) and (1,6).

$\text{Distance} = \sqrt{73}$ (Leave in surd form)

Find the distance between (4,-2) and (1,6) correct to 2 decimal places.

$\text{Distance} = \sqrt{73}$

Distance = 8.54 units (correct to 2 d.p.)

Midpoint Between Two Points

When we are asked to find the midpoint between two points on the number plane, we use the following formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Point 1 (1) Point 2 (2)

Find the midpoint between (-4,1) and (2,3).

$x_1 y_1$ $x_2 y_2$

x of point 1, y of point 1, x of point 2, y of point 2

$$\text{Midpoint} = \left(\frac{-4 + 2}{2}, \frac{1 + 3}{2}\right)$$
$$\text{Midpoint} = \left(\frac{-2}{2}, \frac{4}{2}\right)$$
$$\text{Midpoint} = (-1, 2)$$

Point 1 (1) Point 2 (2)

Find the midpoint between (-4,1) and (2,3).

$x_1 y_1$ $x_2 y_2$

Gradient of a Line Between Two Points

When we are asked to find the gradient of a line between two points on the number plane, we use the following formula:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

Point 1 (1) Point 2 (2)

Find the gradient of the line between (-3,5) and (1,2).

$x_1 y_1$ $x_2 y_2$

x of point 1, y of point 1, y of point 2, x of point 2

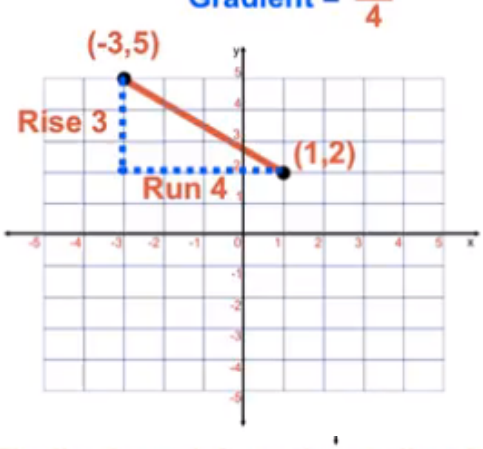
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\text{Gradient} = \frac{2 - 5}{1 - -3}$$
$$\text{Gradient} = \frac{-3}{4}$$

Point 1 (1) Point 2 (2)

Find the gradient of the line between (-3,5) and (1,2).

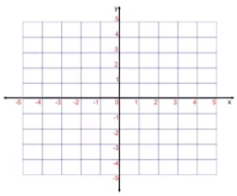
$x_1 y_1$ $x_2 y_2$

$$\text{Gradient} = \frac{-3}{4}$$



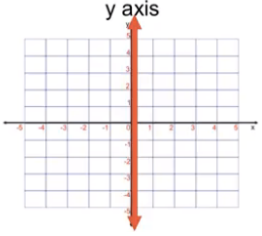
The line leans left, so the gradient is negative.

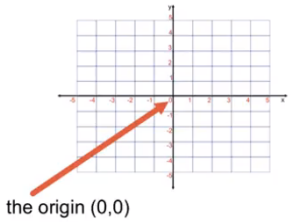
The Number Plane Parts



x axis

y axis





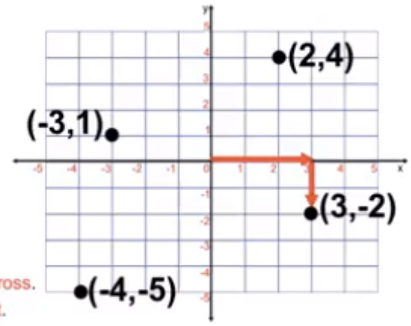
the origin (0,0)

The Number Plane

Starting from the origin,

(3,-2)

right 3, down 2



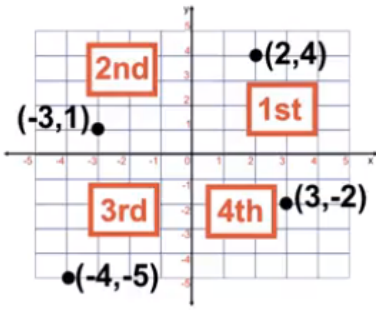
Starting from the origin, (x,y)
x tells us how many places to go across.
A positive x tells us to go to the right.
A negative x tells us to go to the left.

(x,y)

y tells us how many places to go up or down.
A positive y tells us to go to up.
A negative y tells us to go to down.

The Number Plane

Quadrants



Graphing Lines

A linear relationship is formed by a set of points (x,y) that create a straight line when plotted on the number plane.

Graphing Lines Using a Table of Values

$y = 2x - 1$ (Rule or Linear equation)

$y = 2x - 1$

$y = 2(\ \ ) - 1$

x	-1	0	1	2	3
y					

One at a time we sub x values into the rule to find a y value for each x value.

$y = 2x - 1$ (Rule or Linear equation)

$y = 2x - 1$

$y = 2(-1) - 1$

$y = -3$

x	-1	0	1	2	3
y	-3				

x	-1	0	1	2	3
y	-3	-1	1	3	5

+2 +2 +2 +2

At this point, we could just keep the pattern going.

$y = 2x - 1$

x	-1	0	1	2	3
y	-3	-1	1	3	5

(-1,-3) (0,-1) (1,1) (2,3) (3,5)
x y x y x y x y x y

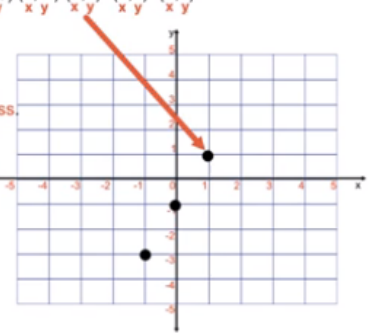
Write each pair of x and y values horizontally.

Plot each pair of x and y values on the number plane.

$y = 2x - 1$

x	-1	0	1	2	3
y	-3	-1	1	3	5

(-1,-3) (0,-1) (1,1) (2,3) (3,5)
x y x y x y x y x y



Starting from the origin, (x,y)
x tells us how many places to go across.
A positive x tells us to go to the right.
A negative x tells us to go to the left.

(x,y)

y tells us how many places to go up or down.
A positive y tells us to go to up.
A negative y tells us to go to down.

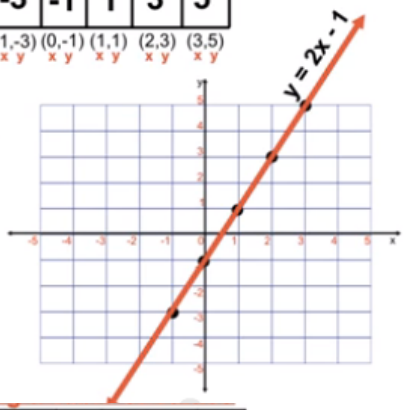
Join the dots...

$y = 2x - 1$

x	-1	0	1	2	3
y	-3	-1	1	3	5

(-1,-3) (0,-1) (1,1) (2,3) (3,5)

Label the line...



$y = 2x - 1$

x	-1	0	1	2	3
y	-3	-1	1	3	5

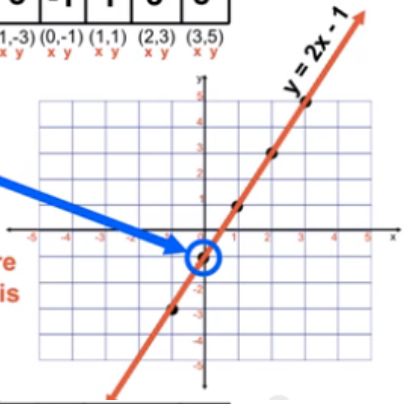
(-1,-3) (0,-1) (1,1) (2,3) (3,5)

Notice how...

$y = 2x - 1$

y intercept = -1

The y intercept is where the line meets the y axis



$y = 2x - 1$

x	-1	0	1	2	3
y	-3	-1	1	3	5

(-1,-3) (0,-1) (1,1) (2,3) (3,5)

Notice how...

$y = \frac{2}{1}x - 1$

We would say that the gradient of this line is $\frac{2}{1}$. $\frac{\text{rise } 2}{\text{run } 1}$

gradient of a line = $\frac{\text{rise}}{\text{run}}$

y = mx + b

A straight line can be in the form of

$y = mx + b$

b is the y intercept
(where the line meets the y axis)

$y = mx + b$

m is the gradient or slope of the line

y = mx + b

$y = mx + b$

m is the gradient or slope of the line

When m is in fraction form,

m tells us the $\frac{\text{rise}}{\text{run}}$ of the line.

When m is positive, the line leans right.

\+

the line is increasing

When m is negative the line leans left.

\-

the line is decreasing

y = mx + b

$y = mx + b$

b is the y intercept (where the line meets the y axis)
m is the gradient or slope of the line
When m is in fraction form, m tells us the $\frac{\text{rise}}{\text{run}}$ of the line.
When m is positive, the line leans right.
When m is negative, the line leans left.

$y = 2x - 1$

This is the y intercept
(the line meets the y axis at -1)

This is the gradient or slope of the line

$2 = \frac{2}{1}$ $\frac{\text{rise}}{\text{run}}$

This is positive, so the line leans right.

y = mx + b

$y = mx + b$

b is the y intercept (where the line meets the y axis)
m is the gradient or slope of the line
When m is in fraction form, m tells us the $\frac{\text{rise}}{\text{run}}$ of the line.
When m is positive, the line leans right.
When m is negative, the line leans left.

$y = \frac{-3}{4}x + 2$

This is the y intercept
(the line meets the y axis at +2)

This is the gradient or slope of the line

$\frac{-3}{4}$ $\frac{\text{rise}}{\text{run}}$

This is negative, so the line leans left.

y = mx + b

$y = mx + b$

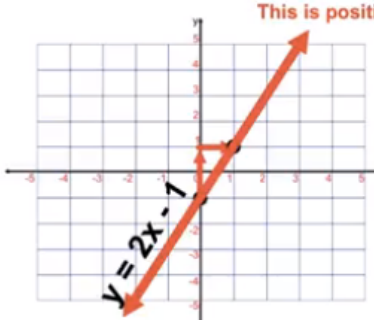
b is the y intercept (where the line meets the y axis)
m is the gradient or slope of the line
When m is in fraction form, m tells us the $\frac{\text{rise}}{\text{run}}$ of the line.
When m is positive, the line leans right.
When m is negative, the line leans left.

$y = 2x - 1$

The line meets the y axis at -1

The gradient is $2 = \frac{2}{1}$ $\frac{\text{rise}}{\text{run}}$

This is positive, so the line leans right.



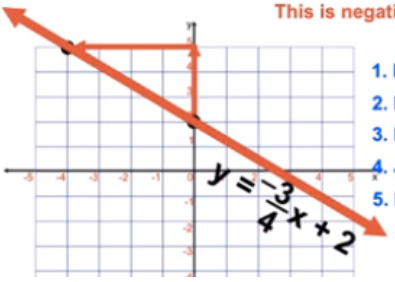
1. Plot the y intercept of -1
2. From there, do the rise & run
3. Lean the line to the right.
4. Join the dots
5. Label the line

y = mx + b

b is the y intercept (where the line meets the y axis)
m is the gradient or slope of the line
When m is in fraction form, m tells us the $\frac{\text{rise}}{\text{run}}$ of the line.
When m is positive, the line leans right.
When m is negative, the line leans left.

$$y = \frac{-3}{4}x + 2$$

The line meets the y axis at +2
The gradient is $\frac{-3 \text{ rise}}{4 \text{ run}}$
This is negative, so the line leans left.



1. Plot the y intercept of +2
2. From there, do the rise & run
3. Lean the line to the left.
4. Join the dots
5. Label the line

X & Y Intercepts

Sketch $y = 2x - 4$

1. Find x intercept (Sub in y = 0)

x intercept happens when y = 0

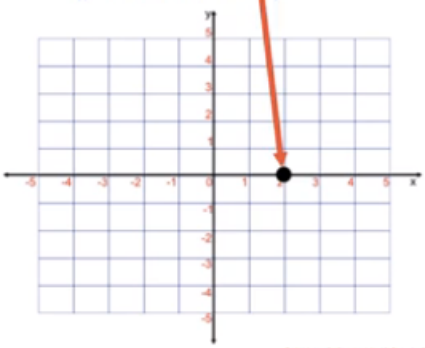
$y = 2x - 4$
$0 = 2x - 4$ (Sub in y = 0)
$+4 \quad +4$
$4 = 2x$ (Solve)
$\div 2 \quad \div 2$
$2 = x$
$x = 2$ (x intercept = 2)

y intercept happens when x = 0

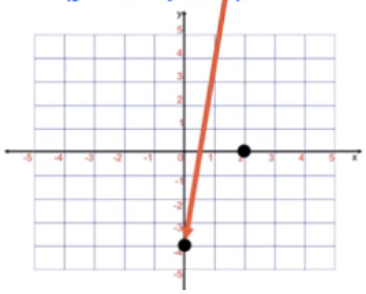
$y = 2x - 4$
$y = 2(0) - 4$ (Sub in x = 0)
$y = -4$ (y intercept = -4)

3. Plot x

(x intercept = 2)
(y intercept = -4)



(x intercept = 2) 4. Plot y intercept
(y intercept = -4)

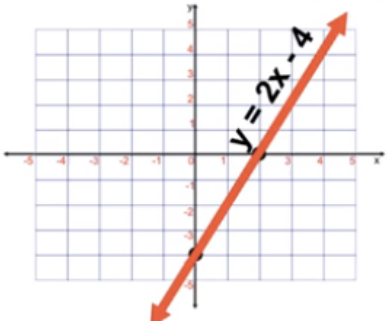


X & Y Intercepts

Sketch $y = 2x - 4$

(x intercept = 2)
(y intercept = -4)

1. Find x intercept (Sub in y = 0)
2. Find y intercept (Sub in x = 0)
3. Plot x intercept
4. Plot y intercept
5. Join the dots
6. Label the line



Sketching Lines Using X & Y Intercepts

1. Find x intercept (Sub in y = 0)
2. Find y intercept (Sub in x = 0)
3. Plot x intercept
4. Plot y intercept
5. Join the dots
6. Label the line

Sketching Lines From Other Equations

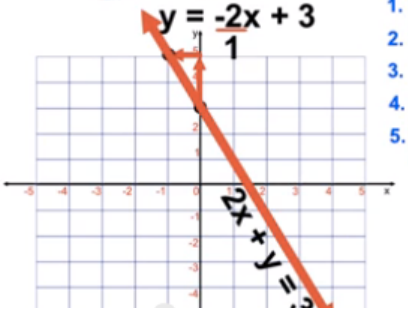
We can often be asked to sketch lines when the linear equation is not in the form of y = mx + b.

Sketch $2x + y = 3$

Method 1
Rearrange into y = mx + b

$2x + y = 3$
$-2x \quad -2x$
$y = -2x + 3$
1

1. Plot the y intercept of +3
2. From there, do the rise & run
3. Lean the line to the left.
4. Join the dots
5. Label the line in the original form



Sketching Lines From Other Equations

Sketch $2x + y = 3$

Method 2
Find x and y intercepts

1. Find x intercept (Sub in y = 0)
2. Find y intercept (Sub in x = 0)

x intercept happens when y = 0

$2x + y = 3$
$2x + (0) = 3$
$2x = 3$
$x = \frac{3}{2}$

(x intercept = $1\frac{1}{2}$)

y intercept happens when x = 0

$2x + y = 3$
$2(0) + y = 3$
$y = 3$

(y intercept = 3)

Sketching Lines From Other Equations

Sketch

$2x + y = 3$

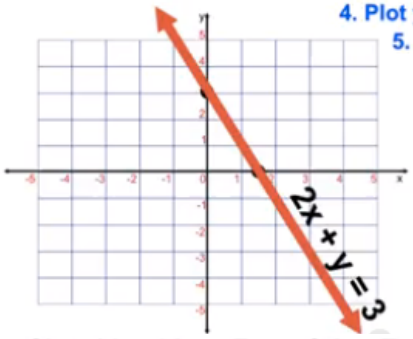
(x intercept = $1\frac{1}{2}$)

(y intercept = 3)

Method 2

Find x and y intercepts

1. Find x intercept (Sub in y = 0)
2. Find y intercept (Sub in x = 0)
3. Plot x intercept
4. Plot y intercept
5. Join the dots
6. Label the line



Sketching Lines From Other Equations

Sketch

$2x + y = 3$

Method 1

Rearrange into y = mx + b

1. Plot the y intercept
2. From there, do the rise & run
3. Lean the line to the right or left.
4. Join the dots
5. Label the line in the original form

Method 2

Find x and y intercepts

1. Find x intercept (Sub in y = 0)
2. Find y intercept (Sub in x = 0)
3. Plot x intercept
4. Plot y intercept
5. Join the dots
6. Label the line

Sketching Lines From Other Equations

Sketch

$2x + 3y = 6$

$-2x \quad -2x$

$3y = -2x + 6$

$\div 3 \quad \div 3 \quad \div 3$

$y = -\frac{2}{3}x + 2$

Method 1

Rearrange into y = mx + b

Move terms to get y on its own.

We move terms by doing the opposite operation to both sides.

Sketching Lines From Other Equations

Sketch

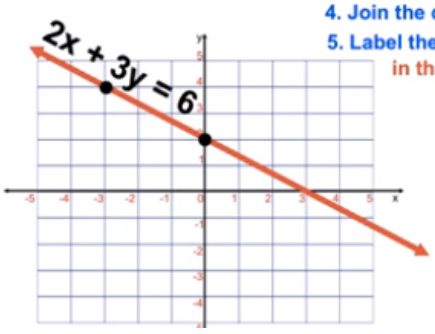
$2x + 3y = 6$

$y = -\frac{2}{3}x + 2$

Method 1

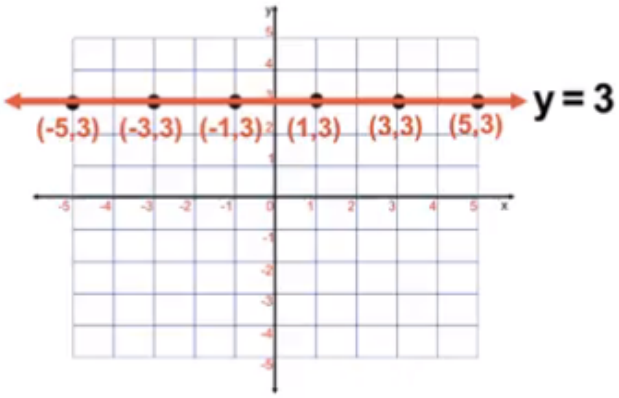
Rearrange into y = mx + b

1. Plot the y intercept
2. From there, do the rise & run
3. Lean the line to the right or left.
4. Join the dots
5. Label the line in the original form



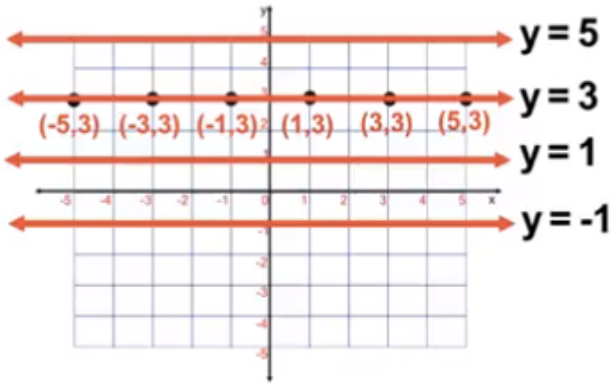
Horizontal & Vertical Lines

Horizontal lines are formed by a set of points that each have the same y value.



Horizontal & Vertical Lines

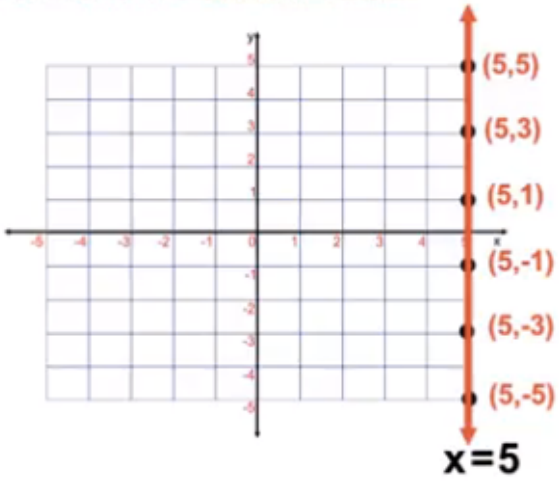
Horizontal lines are formed by a set of points that each have the same y value.



Gradient = $\frac{\text{rise}}{\text{run}} \rightarrow \frac{\text{zero}}{\text{infinite}} \rightarrow$ zero

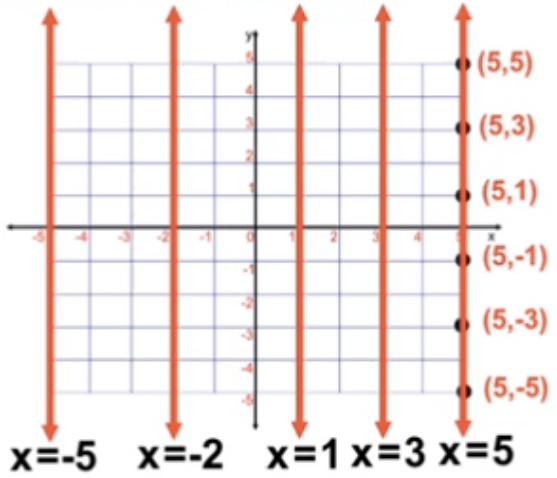
Horizontal & Vertical Lines

Vertical lines are formed by a set of points that each have the same x value.



Horizontal & Vertical Lines

Vertical lines are formed by a set of points that each have the same x value.



Vertical lines are considered to have a gradient that is undefined.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$\rightarrow \frac{\text{infinite}}{\text{zero}}$$

$$\rightarrow \text{undefined}$$

The bottom of a fraction cannot be zero.

A Point Lies on a Line

A point lies on a line if the x and y values of the point make the equation of the line "work properly".

An equation works properly if the LHS = RHS.

Another way of saying that the x and y values make the equation of the line "work properly" is to say that the point satisfies the equation.

Does the point (2,6) lie on the line y = 2x + 2?

(x y)

$y = 2x + 2$

$6 = 2_x2 + 2$

$6 = 4 + 2$

$6 = 6$

LHS = RHS.

Yes! (2,6) does lie on the line y = 2x + 2

Does line y = 4x - 3 pass through the point (5,-2) ?

(x y)

$y = 4x - 3$

$-2 = 4_x5 - 3$

$-2 = 20 - 3$

$-2 = 17$

LHS ≠ RHS

No! The line y = 4x - 3 does not pass through (5,-2)

Does line y = 4x - 3 pass through the point (5,-2) ?

(x y)

$y = 4x - 3$

$-2 = 4_x5 - 3$

$-2 = 20 - 3$

$-2 = 17$

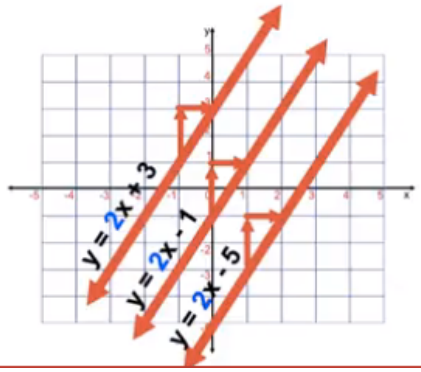
LHS ≠ RHS

No! The line y = 4x - 3 does not pass through (5,-2)

Parallel Lines

Parallel Lines have the same gradient as each other.

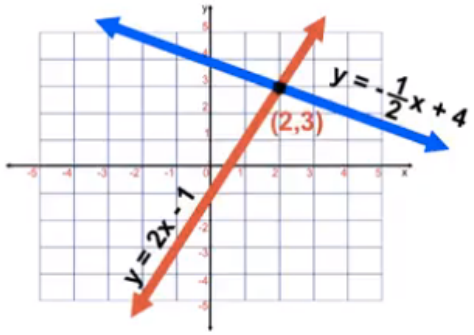
Parallel Lines have the same $\frac{\text{rise}}{\text{run}}$ as each other.



Points of Intersection

A point of intersection is the point where 2 lines meet.

The x and y values of a point of intersection satisfy the equations of both lines at the same time.



Points of Intersection

The x and y values of a point of intersection satisfy the equations of both lines at the same time.

$y = 2x - 1$

$3 = 2_x2 - 1$

$3 = 4 - 1$

$3 = 3$

LHS = RHS

(2,3)

(x y)

$y = -\frac{1}{2}x + 4$

$3 = -\frac{1}{2}{}_x2 + 4$

$3 = -1 + 4$

$3 = 3$

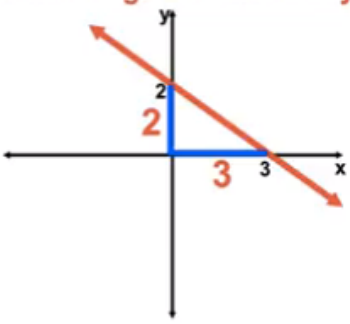
LHS = RHS

The x and y values of the point (2,3) have made both linear equations work properly. → (LHS = RHS)

This shows that (2,3) is the point of intersection of $y = 2x - 1$ and $y = -\frac{1}{2}x + 4$

Finding the Equation From Intercepts

We can find the equation of a line
if we are given the x and y intercepts.



$y = mx + b$

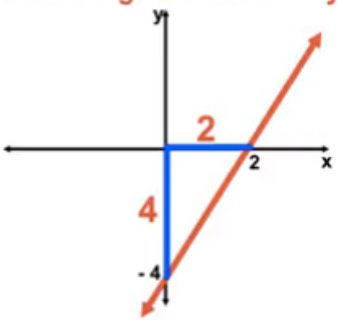
$y = -\frac{2}{3}x + 2$

3 things to find

1. y intercept = 2
2. $\frac{\text{rise}}{\text{run}} = \frac{2}{3}$
3. leans to the left
   m is negative
   m is –

Finding the Equation From Intercepts

We can find the equation of a line
if we are given the x and y intercepts.



$y = mx + b$

$y = 2x - 4$

3 things to find

1. y intercept = - 4
2. $\frac{\text{rise}}{\text{run}} = \frac{4}{2} \rightarrow \frac{2}{1}$
3. lean to the right
   m is positive
   m is +

Finding the Equation from the Gradient & a Point

Find the equation of a line with a gradient of 2 (m),
if the line passes through the point (1,6) (x y).

$y = mx + b$
$6 = 2_x 1 + b$
$6 = 2 + b$
$-2 \quad -2$
$4 = b$
$b = 4$

1. Substitute all values into $y = mx + b$
2. Solve to get b on its own.
3. Write the final version of $y = mx + b$

$y = 2x + 4$

Finding an Equation From a Table of Values

Find the linear equation for the table of values below:

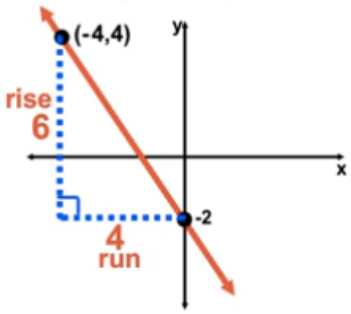
x	-1	0	1	2	3
y	-3	-1	1	3	5

+2 +2 +2 +2

$y = mx + b$
$y = 2x + b$

Finding the Equation of a Line From a Sketch

Find the equation of the line:



$y = mx + b$

$y = -\frac{3}{2}x - 2$

3 things to find

1. y intercept = - 2
2. $\frac{\text{rise}}{\text{run}} = \frac{6}{4} \rightarrow \frac{3}{2}$
3. leans to the left
   m is negative
   m is –

x	-1	0	1	2	3
y	-3	-1	1	3	5

+2 +2 +2 +2

$y = mx + b$
$y = 2x + b$

The y intercept of a line occurs when x = 0.
The y intercept of this line is -1.

$y = mx + b$

$y = 2x - 1$

Alternate way to find y intercept:
Adjusting from $y = 2x$

$5 = 2_x 3$
$5 = 6 - 1$

$y = mx + b$

$y = 2x - 1$

Alternate way to find y intercept:
Substitute in a pair of values into
$y = mx + b$
$5 = 2_x 3 + b$
$5 = 6 + b$
$-6 \quad -6$
$-1 = b \rightarrow b = -1$