EPPING BOYS HIGH SCHOOL

YEAR 12 2 UNIT MATHEMATICS

2012 HSC EXAMINATION – TRIAL PAPER

Student Number: ....

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen. Black is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in Questions 11 – 16

Time Allowed: 3 hours

Total marks – 100

Section I: Multiple Choice
Questions 1 – 10 10 marks

- Attempt all questions
- Answer on the Answer Sheet provided
- Allow about 15 minutes for this section

Section II: Extended Response
Questions 11 – 16 90 marks

- Attempt all questions
- Allow about 2 hours 45 minutes for this section

Class Teacher:

(Please tick, highlight or circle)

O Mr Bui – 12MX1-1

O Mr McKenzie – 12MX1-2

O Mr Martin – 12MX1-3

O Ms Tang – 12M4

O Ms Dhillon – 12M5

	Section 1	Section 2						Total
	MC	Q11	Q12	Q13	Q14	Q15	Q16	
Out of	10	15	15	15	15	15	15	100
Mark								

Section I

Questions 1 – 10 (1 mark for each question)

Read each question and choose an answer A, B, C or D
Record your answer on the Answer Sheet provided.
Allow about 15 minutes for this section.

1. Evaluate

$$\sum_{n=4}^{10} 5n + 2$$

A 132

B 222

C 259

D 154

2. What is the derivative of $y = \sin^2 4x$ with respect to x?

A $y' = 8 \sin 4x \, cos \, 4x$

B $y' = -8 \sin 4x \, cos \, 4x$

C $y' = 2 \sin 4x \, cos \, 4x$

D $y' = -2 \sin 4x \, cos \, 4x$

3. Find

$$\int (e^x + 2)^2 \, dx$$

A $\frac{1}{2}e^{2x} + 4e^x + 4x + C$

B $2e^{2x} + 4e^x + 4x + C$

C $\frac{(e^x + 2)^3 . e^x}{3} + C$

D $\frac{1}{3}e^{3x} + 4e^x + 4x + C$

4. Solve $2x^2 + 5x + 2 \geq 0$

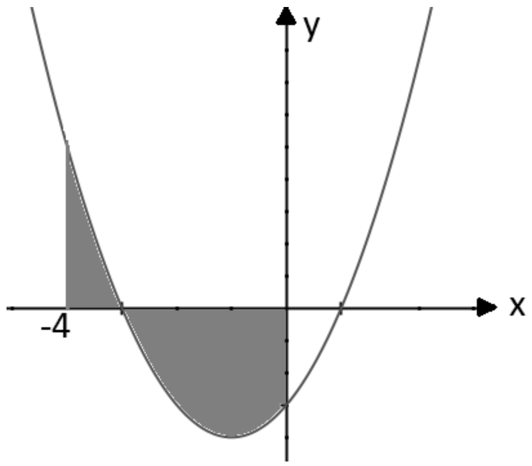
A $x \leq -2, x \leq -\frac{1}{2}$

B $-2 \leq x \leq -\frac{1}{2}$

C $x \leq -2, x \geq -\frac{1}{2}$

D $x \geq -2, x \geq -\frac{1}{2}$

5. Which expression will give the area of the shaded region bounded by the curve $y = x^2 + 2x - 3$, the x-axis and the lines $x = 0$ and $x = -4$



A $A = \left|\int_{-4}^{-3} x^2 + 2x - 3\, dx\right| + \int_{-3}^{0} x^2 + 2x - 3\, dx$

B $A = \int_{-4}^{-3} x^2 + 2x - 3\, dx + \left|\int_{-3}^{0} x^2 + 2x - 3\, dx\right|$

C $A = \left|\int_{-4}^{-3} x^2 + 2x - 3\, dx\right| - \int_{-3}^{0} x^2 + 2x - 3\, dx$

D $A = \int_{-4}^{-3} x^2 + 2x - 3\, dx - \left|\int_{-3}^{0} x^2 + 2x - 3\, dx\right|$

6. If $27^{x+3} = 3^x$, then

A $x = \frac{9}{2}$

B $x = -\frac{9}{2}$

C $x = \frac{3}{2}$

D $x = -\frac{3}{2}$

7. The gradient of the normal to the curve $y = x^2 + 5x - 3$ at the point $(2, 11)$ is

A $m = 9$

B $m = -9$

C $m = \frac{1}{9}$

D $m = -\frac{1}{9}$

8. What are the values of a and b if

$$\frac{2\sqrt{7}}{\sqrt{7}+2} = a + b\sqrt{7}$$

A $a = \frac{14}{3}, b = \frac{4}{3}$

B $a = \frac{14}{3}, b = -\frac{4}{3}$

C $a = 14, b = 4$

D $a = 14, b = -4$

9. The centre and radius given by the circle $x^2 + 2x + y^2 + 4y - 5 = 0$ is

A Centre $= (-1, -2)$ and Radius $= 10$

B Centre $= (1, 2)$ and Radius $= 10$

C Centre $= (-1, -2)$ and Radius $= \sqrt{10}$

D Centre $= (1, 2)$ and Radius $= \sqrt{10}$

10. If $y = \ln(2x^2 + 2)$ then its derivative in simplest form is given by

A $\frac{dy}{dx} = \frac{2x^2 + 2}{4x}$

B $\frac{dy}{dx} = \frac{4x}{2x^2 + 2}$

C $\frac{dy}{dx} = \frac{2x}{x^2 + 2}$

D $\frac{dy}{dx} = \frac{2x}{x^2 + 1}$

Section II

Questions 11 – 16 (15 marks each)

Allow about 2 hours 45 minutes for this section.

Question 11 **MARKS**

a. Find the derivative for each of the following, simplifying answers where necessary:

i. $y = xe^{\pi}$ 1

ii. $y = cos(e^{-x})$ 2

iii. $y = \ln(x^3 + 4x)$ 2

b. Evaluate 2

$$\int_0^{\frac{\pi}{4}} (\sin 2x + \sec^2 x)\, dx$$

c. Write down the exact value of 2

$$\cot\left(\frac{5\pi}{6}\right)$$

d. Show that

$$\sec\theta + \tan\theta = \frac{1 + \sin\theta}{\cos\theta}$$ 2

e. Find the gradient of the normal to the curve $y = x^3 - 4x - 1$ at the point $T(2, -1)$ 2

f. Find the coordinates of the vertex of the parabola with equation 2

$$x^2 - 10x - 16y - 7 = 0$$

Question 12 **MARKS**

a. Use the limiting sum formula to rewrite the recurring decimal, $0.\dot{2}\dot{5}$ in simplest fraction form 2

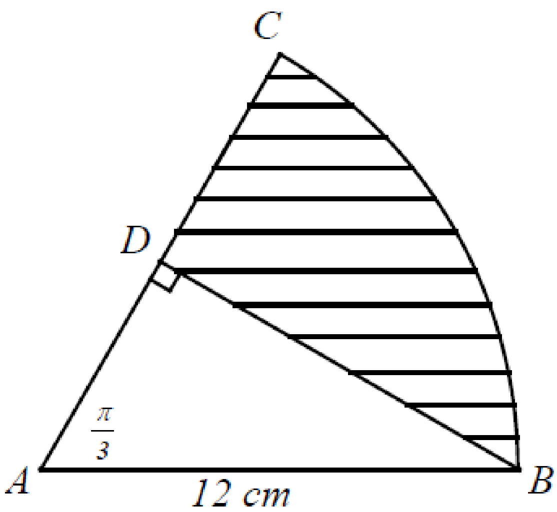
b. If α and β are roots of the equation $3x^2 - 12x - 9 = 0$, find the value of

i. $\alpha^2 + \beta^2$ 2

ii. Hence, evaluate 2

$$\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$$

c.



ABC is a sector with $\angle BAC = \frac{\pi}{3}$ and $AC = AB = 12\ cm$

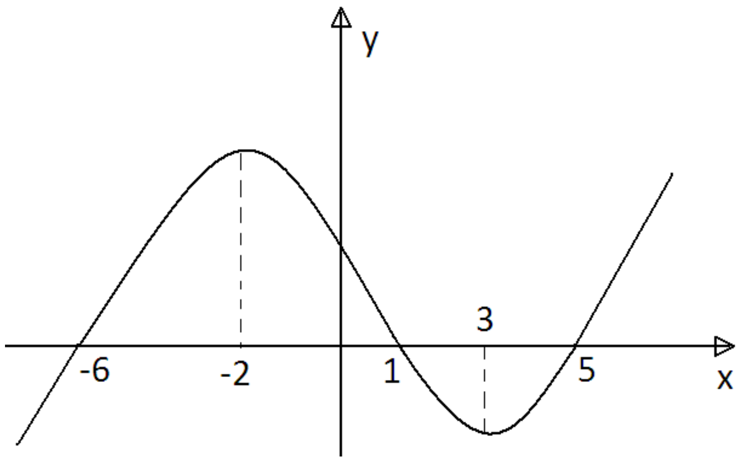
i. Calculate the area of sector ABC 1

ii. Calculate the area of triangle ABD 2

iii. Calculate the area of the shaded region 1

d. Given that $(4 - \sqrt{5})^2 = a - \sqrt{b}$, find the values of a and b where a and b are rational 2

e. The diagram below shows the graph of a function $y = f(x)$

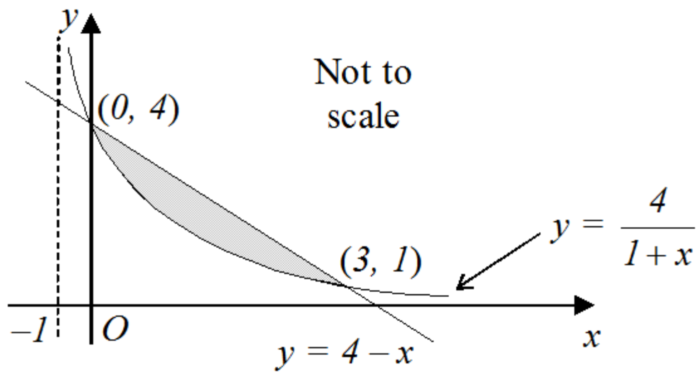


i. For which values of x is the derivative, $f'(x)$ negative 1

ii. Sketch the graph $y = f'(x)$ 2

Question 13 MARKS

a. The diagram below shows part of the hyperbola $y = \frac{4}{1+x}$ and the line $y = 4 - x$



i. Show that the line $y = 4 - x$ intersects the hyperbola $y = \frac{4}{1+x}$ at $(0,4)$ and $(3,1)$ 2

ii. Hence calculate the exact area of the shaded region. 2

b. Solve $2\cos 2x - \sqrt{3} = 0$ in the domain $0 \leq x \leq 2\pi$, leaving your answers in exact form 3

c. Find the derivative of $y = (3x^2 + 1)^5(4 - x)^4$ in factorised form 3

d. A line L is inclined at an angle of 45° to the positive direction of the x-axis and passes through the point $X(\sqrt{2}, -1)$

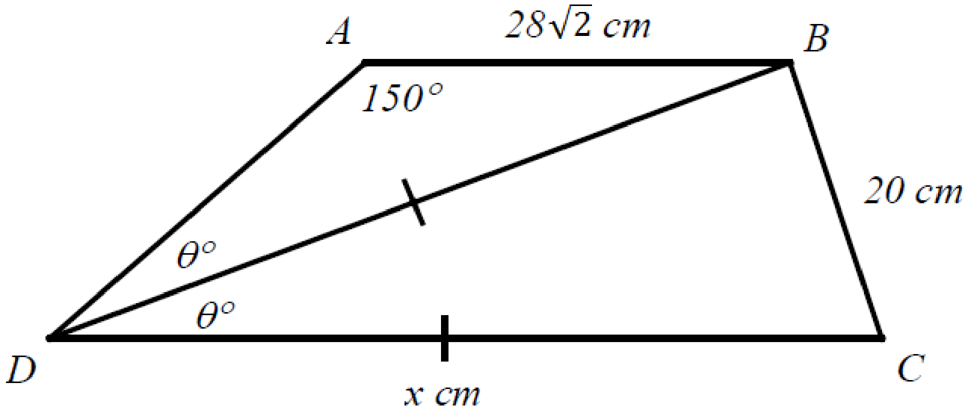
i. Find the equation (in the exact form) of the line L 2

ii. Show that $P(\sqrt{2} + 1, 0)$ and $Q(0, -\sqrt{2} - 1)$ are the points of intersection with the x and y axes respectively, of the line L 2

iii. Hence or otherwise, find the exact area of triangle OPQ where O is the origin 1

Question 14 MARKS

a. In the quadrilateral $ABCD$ shown below, the diagonal BD bisects $\angle ADC$ and is equal to the side DC



Given $AB = 28\sqrt{2}\ cm, BC = 20\ cm$ and $\angle DAB = 150°$.
Let $DC = x$ and $\angle BDC = \theta°$

i. Show that 2

$$\sin\theta = \frac{14\sqrt{2}}{x}$$

ii. By considering the triangle BDC, find an expression for $\cos\theta$ in terms of x 2

b. Consider the curve $y = x^3 - 2x^2 - 3x$

i. Find the x and y intercepts 2

ii. Find the coordinates of the turning points of the curve and determine their nature 2

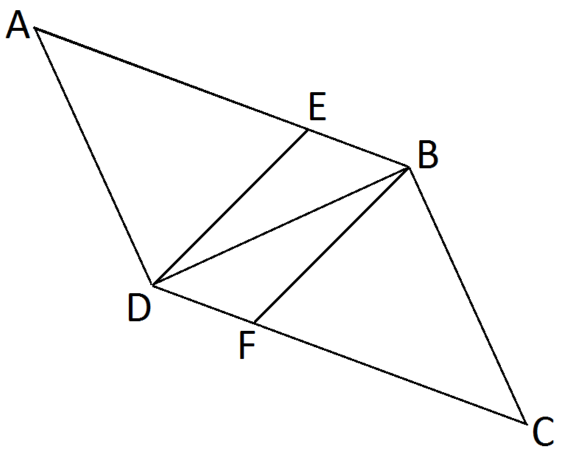
iii. Find the coordinates of the point of inflexion 2

iv. Sketch the curve showing all the turning points, the point of inflexion and the x and y intercepts 2

c. Calculate the exact volume generated when the region enclosed by the curve $y = 2 + 4e^{-2x}$ for $0 \leq x \leq 1$ is rotated about the x-axis 3

Question 15 **MARKS**

a. In the parallelogram $ABCD$, E lies on the side AB and F lies on the side CD such that $AE = CF$. 2



Copy the diagram into your answer booklet showing all given information.
Prove that $\Delta BED \equiv \Delta DFB$

b. Using Simpson's Rule with 5 function values, evaluate the following correct to 4 decimal places 3

$$\int_0^1 x \cos x \; dx$$

c. Diana borrows \$90 000 at 15% pa interest for 25 years and agrees to repay the loan in equal monthly instalments

i. Calculate the value of each monthly instalment 3

ii. What will be the total amount of interest paid on this loan? 1

d. Given that $f(x) = x^2\sqrt{10-x}$,

i. Show that 2

$$f'(x) = \frac{5x(8-x)}{2\sqrt{10-x}}$$

ii. Hence, find all the stationary points on the curve $y = x^2\sqrt{10-x}$ and determine their nature 4

Question 16 **MARKS**

a. Find the value of

$$\lim_{x \to 0}\left(\frac{\sin 7x}{5x}\right)$$ 1

b. Sam was born on the 1st of January. On the day he was born, his father opened a trust account by depositing \$220. Each year on Sam's birthday, his father deposited \$220 into this trust account and continued to do this up to and including his 20th birthday. When Sam turned 21, his father collected the total amount and presented it to him. This trust account paid an interest of 6% pa compounded every year. How much did Sam receive on his 21st birthday? 3

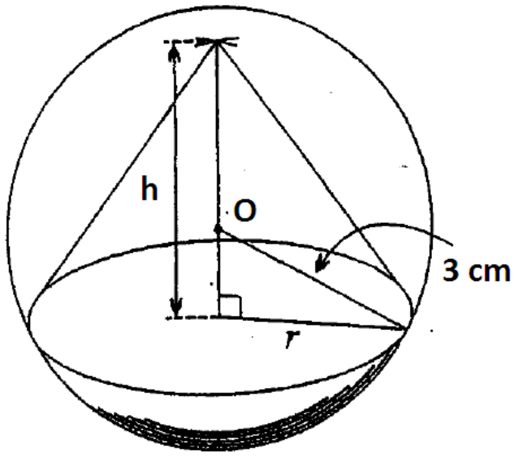
c. Sketch the curve of $y = 1 - \sin 2x$ for $0 \leq x \leq 2\pi$, showing all important features 2

d. x, x^2 and $5x$ are three consecutive terms of an arithmetic series

i. Show that $2x^2 - 6x = 0$ 2

ii. What is the common difference of this arithmetic series? 2

e. A right circular cone of height h cm and base radius r cm is inscribed in a sphere of radius 3 cm, as shown below.



i. Show that the volume (V) of the cone is given by the formula 2

$$V = \frac{\pi}{3}\,(6h^2 - h^3)$$

ii. Find the height of the cone so that its volume is maximised 3

– THE END OF THE EXAMINATION ☺ –

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Student Number: ……………………………………….......

Teacher: ……………………………………………………..........

MULTIPLE CHOICE ANSWER SHEET

INSTRUCTIONS:

- Remember to write your name and your teacher's name on the top
- Cross the box that indicates the correct answer

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

EPPING BOYS HIGH SCHOOL

2U MATHEMATICS
2012 HSC TRIAL PAPER SOLUTIONS

SECTION I – MULTIPLE CHOICE

1.

$$\sum_{n=4}^{10} 5n+2$$
$= 22+27+32+\cdots+52$
$= \frac{7}{2}(22+52)$
$= 259$
C

2. $y = \sin^2 4x$
$y = (\sin 4x)^2$
$y' = 2\sin 4x \times 4\cos 4x$
$\therefore y' = 8\sin 4x \cos 4x$
A

3.

$$\int (e^x+2)^2\,dx$$
$$= \int e^{2x}+4e^x+4\,dx$$
$$= \frac{1}{2}e^{2x}+4e^x+4x+C$$
A

4. $2x^2+5x+2 \geq 0$
$(2x+1)(x+2) \geq 0$
$x \leq -2, x \geq -\frac{1}{2}$
C

5. $y = x^2+2x-3$
$y = 0; (x-1)(x+3) = 0 \rightarrow x = 1,-3$
$$A = \int_{-4}^{-3} x^2+2x-3\,dx + \left|\int_{-3}^{0} x^2+2x-3\,dx\right|$$
B

6. $27^{x+3} = 3^x$
$3^{3x+9} = 3^x$
$3x+9 = x$
$2x = -9$
$x = -\frac{9}{2}$
B

7. $y = x^2+5x-3$
$\frac{dy}{dx} = 2x+5$
When $x = 2, m_T = 9$
$\therefore m_N = -\frac{1}{9}$
D

8.

$$\frac{2\sqrt{7}}{\sqrt{7}+2} = a+b\sqrt{7}$$
$$\frac{2\sqrt{7}}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2} = \frac{14-4\sqrt{7}}{3}$$
$\therefore a = \frac{14}{3}, b = -\frac{4}{3}$
B

9. $x^2+2x+y^2+4y-5 = 0$
$x^2+2x+1+y^2+4y+4 = 10$
$(x+1)^2+(y+2)^2 = 10$
Centre $= (-1,-2)$ and Radius $= \sqrt{10}$
C

10. $y = \ln(2x^2+2)$
$\frac{dy}{dx} = \frac{4x}{2x^2+2}$
$\frac{dy}{dx} = \frac{2x}{x^2+1}$
D

SECTION II – FREE RESPONSE

QUESTION 11

a.

i. $y = xe^\pi$
$\frac{dy}{dx} = e^\pi$

ii. $y = \cos(e^{-x})$
$\frac{dy}{dx} = -e^{-x} \times -\sin(e^{-x})$
$\frac{dy}{dx} = e^{-x}\sin(e^{-x})$

iii. $y = \ln(x^3+4x)$
$\frac{dy}{dx} = \frac{3x^2+4}{x^3+4x}$

b.

$$\int_0^{\frac{\pi}{4}} (\sin 2x + \sec^2 x)\, dx$$
$$= \left[-\frac{\cos 2x}{2} + \tan x\right]_0^{\frac{\pi}{4}}$$
$$= \left(-\frac{1}{2}\cos\frac{\pi}{2} + \tan\frac{\pi}{4}\right) - \left(-\frac{1}{2}\cos 0 + \tan 0\right)$$
$$= 0 + 1 + \frac{1}{2} - 0$$
$$= \frac{3}{2}$$

c.

$$\cot\left(\frac{5\pi}{6}\right)$$
$$= \frac{1}{\tan\left(\frac{5\pi}{6}\right)}$$
$$= \frac{1}{-\tan\left(\frac{\pi}{6}\right)}$$
$$= \frac{1}{-\frac{1}{\sqrt{3}}}$$
$$= -\sqrt{3}$$

d. Show

$$\sec\theta + \tan\theta = \frac{1 + \sin\theta}{\cos\theta}$$

LHS

$$= \sec\theta + \tan\theta$$
$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$
$$= \frac{1 + \sin\theta}{\cos\theta}$$

= RHS

$$\therefore \sec\theta + \tan\theta = \frac{1 + \sin\theta}{\cos\theta}$$

e. $y = x^3 - 4x - 1$

$$\frac{dy}{dx} = 3x^2 - 4$$

When $x = 2$

$$m_T = 3(2^2) - 4 = 8$$
$$\therefore m_N = -\frac{1}{8}$$

f. $x^2 - 10x - 16y - 7 = 0$

$$x^2 - 10x = 16y + 7$$
$$x^2 - 10x + 25 = 16y + 32$$
$$(x - 5)^2 = 16(y + 2)$$

$\therefore$ Vertex $= (5, -2)$

QUESTION 12

a. $0.\dot{2}\dot{5}$

$$= \frac{25}{100} + \frac{25}{10\,000} + \frac{25}{1000\,000} + \cdots$$
$$= \frac{\frac{25}{100}}{1 - \frac{1}{100}}$$
$$= \frac{25}{100} \div \frac{99}{100}$$
$$= \frac{25}{99}$$

b. $3x^2 - 12x - 9 = 0$

i. $\alpha^2 + \beta^2$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$
$$= 4^2 - 2(-3)$$
$$= 22$$

ii.

$$\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$$
$$= \frac{\beta^2 + \alpha^2}{\alpha\beta}$$
$$= \frac{22}{-3}$$

c.

i. Area of sector ABC

$$= \frac{1}{6} \times \pi \times 12^2$$

$= 24\pi\ cm^2$

$= 75.4\ cm^2$ (to 1 dec. pl.)

ii. $AD = 12 \times \cos\frac{\pi}{3} = 6\ cm$

Area of triangle ABD

$$= \frac{1}{2} \times 6 \times 12 \times \sin\frac{\pi}{3}$$
$$= 18\sqrt{3}\ cm^2$$

iii. Area of shaded region

$= 24\pi - 18\sqrt{3}$

$= 44.2\ cm^2$ (to 1 dec. pl.)

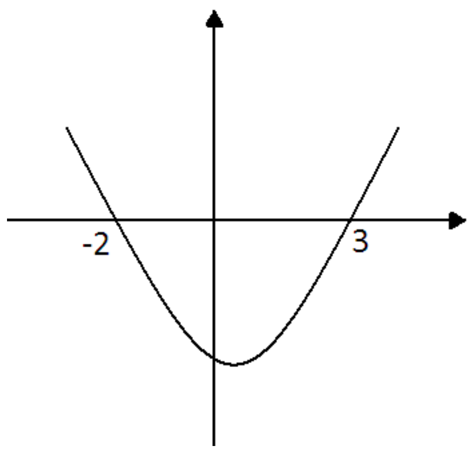
d. $(4 - \sqrt{5})^2 = a - \sqrt{b}$

$$(4 - \sqrt{5})^2$$
$$= 16 - 8\sqrt{5} + 5$$
$$= 21 - \sqrt{320}$$
$$\therefore a = 21, b = 320$$

e.

i. $f'(x)$ is negative when $-2 < x < 3$
($\frac{1}{2}$ mark was awarded if answer was put as $-2 \leq x \leq 3$)

ii.



QUESTION 13

a.

i. Solve the two equations simultaneously

$y = 4 - x \quad (1)$

$y = \frac{4}{1+x} \quad (2)$

Sub (1) into (2)

$4 - x = \frac{4}{1+x}$

$(4-x)(1+x) = 4$

$4 + 4x - x - x^2 = 4$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3 \quad (3)$

Sub (3) into (1)

$y = 4 - 0 = 4$

$y = 4 - 3 = 1$

$\therefore x = 0, y = 4 \rightarrow (0, 4)$

$x = 3, y = 1 \rightarrow (3, 1)$

ii. Area of the shaded region

$$= \int_0^3 4 - x - \left(\frac{4}{1+x}\right) dx$$

$$= \left[4x - \frac{x^2}{2} - 4\ln(1+x)\right]_0^3$$

$$= 12 - \frac{9}{2} - 4\ln 4 - (0 - 4\ln 1)$$

$$= \left(\frac{15}{2} - 4\ln 4\right) units^2$$

b. $2\cos 2x - \sqrt{3} = 0$

$\cos 2x = \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\frac{\sqrt{3}}{2}$

$2x = 30°, 330°, 390°, 690°$

$x = 15°, 165°, 195°, 345°$

$\therefore x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

c. $y = (3x^2+1)^5(4-x)^4$

$u = (3x^2+1)^5 \rightarrow u' = 30x(3x^2+1)^4$

$v = (4-x)^4 \rightarrow v' = -4(4-x)^3$

$\therefore \frac{dy}{dx}$

$= -4(4-x)^3(3x^2+1)^5 + 30x(3x^2+1)^4(4-x)^4$

$= 2(3x^2+1)^4(4-x)^3[-2(3x^2+1) + 15x(4-x)]$

$= 2(3x^2+1)^4(4-x)^3(-2 + 60x - 21x^2)$

d.

i. $m = \tan 45° = 1, (\sqrt{2}, -1)$

$y + 1 = 1(x - \sqrt{2})$

$y = x - \sqrt{2} - 1$

ii. $y = x - \sqrt{2} - 1$

When $x = 0, y = -\sqrt{2} - 1$

$\therefore Q = (0, -\sqrt{2} - 1)$

When $y = 0, y = \sqrt{2} + 1$

$\therefore P = (\sqrt{2} + 1, 0)$

iii. Area of triangle OPQ

$= \left|\frac{1}{2} \times (\sqrt{2}+1) \times (-\sqrt{2}-1)\right|$

$= \left|-\frac{1}{2}(2 + 2\sqrt{2} + 1)\right|$

$= \left|-\frac{1}{2}(3 + 2\sqrt{2})\right|$

$= \frac{3 + 2\sqrt{2}}{2}\ units^2$

QUESTION 14

a.

i.

$\frac{\sin\theta}{28\sqrt{2}} = \frac{\sin 150°}{x}$

$\sin\theta = \frac{28\sqrt{2} \times \sin 150°}{x}$

$\sin\theta = \frac{28\sqrt{2} \times \frac{1}{2}}{x}$

$$\therefore \sin\theta = \frac{14\sqrt{2}}{x}$$

ii.

$$\cos\theta = \frac{x^2 + x^2 - 20^2}{2x^2}$$
$$\cos\theta = \frac{2x^2 - 400}{2x^2}$$
$$\therefore \cos\theta = \frac{x^2 - 200}{x^2}$$

b. $y = x^3 - 2x^2 - 3x$

i. For y intercepts, when $x = 0, y = 0$

For x intercepts, when $y = 0$

$x^3 - 2x^2 - 3x = 0$

$x(x^2 - 2x - 3) = 0$

$x(x-3)(x+1) = 0$

$\therefore x = 0, 3, -1$

ii. $y = x^3 - 2x^2 - 3x$

$$\frac{dy}{dx} = 3x^2 - 4x - 3$$
$$\frac{d^2y}{dx^2} = 6x - 4$$

$$For\ stationary\ points, \frac{dy}{dx} = 0$$

$3x^2 - 4x - 3 = 0$

$$x = \frac{4 \pm \sqrt{52}}{6} = \frac{2 \pm \sqrt{13}}{3}$$
$$x = \frac{2+\sqrt{13}}{3}, \frac{2-\sqrt{13}}{3}$$

$x = 1.87, -0.54$ (2 d.pl.)

When $x = 1.87, y = -6.06$ (2 d.pl.)

When $x = -0.54, y = 0.88$ (2 d.pl.)

For nature

When $x = 1.87$

$$\frac{d^2y}{dx^2} = 7.21 > 0 \ \therefore min$$

When $x = -0.54$

$$\frac{d^2y}{dx^2} = -7.21 < 0 \ \therefore max$$

$\therefore (1.87, -6.06)$ is a minimum turning point and $(-0.54, 0.88)$ is a maximum turning point

iii. For inflexion points

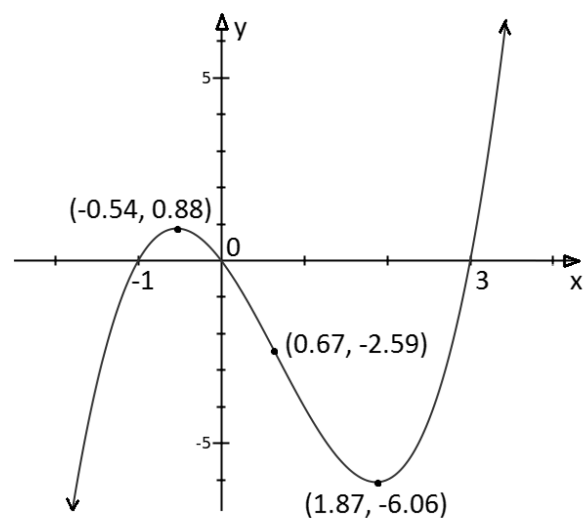
$$\frac{d^2y}{dx^2} = 0$$
$$\frac{d^2y}{dx^2} = 6x - 4$$
$$\therefore 6x - 4 = 0$$
$$\therefore x = \frac{2}{3}$$
$$When\ x = \frac{2}{3},$$
$$y = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right) = -\frac{70}{27}$$

Check concavity

x	0	$\frac{2}{3}$	1
$\frac{d^2y}{dx^2}$	-4	0	2

$$\therefore inflexion\ point\ at\ \left(\frac{2}{3}, -\frac{70}{27}\right)$$

iv.



c. $y = 2 + 4e^{-2x}$

$y^2 = (2 + 4e^{-2x})^2 = 4 + 16e^{-2x} + 16e^{-4x}$

$$V = \pi \int_0^1 4 + 16e^{-2x} + 16e^{-4x}\ dx$$
$$= \pi \left[4x - 8e^{-2x} - 4e^{-4x}\right]_0^1$$
$$= \pi[(4 - 8e^{-2} - 4e^{-4}) - (-8 - 4)]$$
$$= \pi(16 - 8e^{-2} - 4e^{-4})\ units^3$$

QUESTION 15

a. $AB = CD$

(opposite sides of a parallelogram are equal)

$EB = DF$

(found that $AB = CD$ and given that $AE = CF$)

In $\Delta s\ BED$ and DFB

$EB = DF$ (proven above)

DB (common side)

$\angle DBE = \angle BDF$ (alt. $\angle$s, $AB // CD$)

$\therefore \Delta BED \equiv \Delta DFB$ (SAS)

b.

$$\int_0^1 x\cos x\ dx$$

x	0	0.25	0.5	0.75	1
y	0	$\frac{1}{4}\cos\frac{1}{4}$	$\frac{1}{2}\cos\frac{1}{2}$	$\frac{3}{4}\cos\frac{3}{4}$	$\cos 1$

$$= \frac{1}{12}\left(0 + \cos 1 + 4\left(\frac{1}{4}\cos\frac{1}{4} + \frac{3}{4}\cos\frac{3}{4}\right) + 2\left(\frac{1}{2}\cos\frac{1}{2}\right)\right)$$

$$= 0.3818$$

c.

i. Let M = monthly installments

$r = 1.25\% = 0.0125$

$n = 300$

$A_1 = 90000(1.0125) - M$

$A_2 = 90000(1.0125)^2 - M(1.0125) - M$

$A_3 = 90000(1.0125)^3 - M(1.0125)^2 - M(1.0125) - M$

...

$A_{300} = 90000(1.0125)^{300} - M(1.0125)^{299} - M(1.0125) \ldots - M(1.0125) - M$

$A_{300} = 90000(1.0125)^{300} - M(1 + 1.0125 + \cdots + 1.0125^{298} + 1.0125^{299})$

$A_{300} = 90000(1.0125)^{300} - M\left(\frac{1.0125^{300} - 1}{1.0125 - 1}\right)$

Since $A_{300} = 0$, then

$$0 = 90000(1.0125)^{300} - M\left(\frac{1.0125^{300} - 1}{0.0125}\right)$$

$$M\left(\frac{1.0125^{300} - 1}{0.0125}\right) = 90000(1.0125)^{300}$$

$$M = 90000(1.0125)^{300} \div \left(\frac{1.0125^{300} - 1}{0.0125}\right)$$

$$M = \$1152.75$$

ii. Total amount repaid

$= 1152.75 \times 300$

$= \$345\ 825$

Interest paid

$= \$345\ 825 - 90\ 000$

$= \$255\ 825$

d. $f(x) = x^2\sqrt{10 - x}$

i. Let $u = x^2 \rightarrow u' = 2x$

$v = (10 - x)^{\frac{1}{2}} \rightarrow v' = -\frac{1}{2}(10 - x)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = x^2 \times -\frac{1}{2}(10 - x)^{-\frac{1}{2}} + 2x(10 - x)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{-x^2}{2\sqrt{10 - x}} + 2x\sqrt{10 - x}$$

$$\frac{dy}{dx} = \frac{-x^2 + 4x(10 - x)}{2\sqrt{10 - x}}$$

$$\frac{dy}{dx} = \frac{-5x^2 + 40x}{2\sqrt{10 - x}}$$

$$\therefore \frac{dy}{dx} = \frac{5x(8 - x)}{2\sqrt{10 - x}}$$

ii. $y = x^2\sqrt{10 - x}$

$$\frac{dy}{dx} = \frac{5x(8 - x)}{2\sqrt{10 - x}}$$

For stationary points, $\frac{dy}{dx} = 0$

$$\frac{5x(8 - x)}{2\sqrt{10 - x}} = 0$$

$5x(8 - x) = 0$

$\therefore x = 0, 8$

For nature

When $x = 0$

x	-1	0	2
$\frac{dy}{dx}$	-6.78	0	10.61

When $x = 8$

x	7	8	9
$\frac{dy}{dx}$	10.1	0	-22.5

When $x = 0 \rightarrow y = 0$

When $x = 8 \rightarrow y = 64\sqrt{2}$

$\therefore$ (0,0) is a minimum turning point and $(8, 64\sqrt{2})$ is a maximum turning point

QUESTION 16

a.

$$\lim_{x\to 0}\left(\frac{\sin 7x}{5x}\right)$$

$$= \frac{1}{5}\lim_{x\to 0}\left(\frac{\sin 7x}{x}\right)$$

$$= \frac{7}{5}\lim_{x\to 0}\left(\frac{\sin 7x}{7x}\right)$$

$$= \frac{7}{5}$$

b. Given $P = \$220, r = 6\% = 0.06, n = 21$

First amount $= 220(1.06)^{21}$

Second amount $= 220(1.06)^{20}$

Third amount $= 220(1.06)^{19}$

...

Last amount $= 220(1.06)$

Total amount

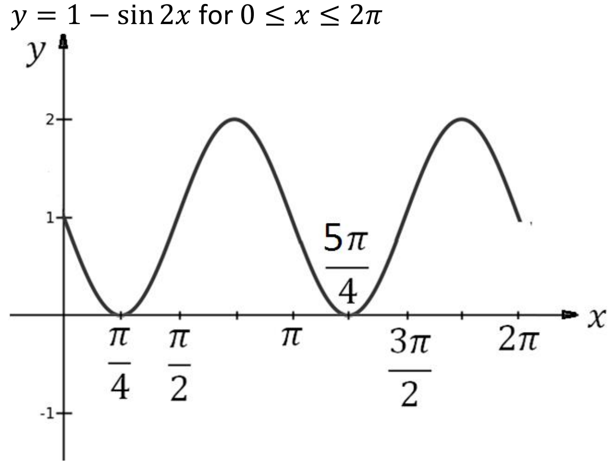
$= 220(1.06)^{21} + 220(1.06)^{20} + \cdots + 220(1.06)$

$= 220(1.06) + \cdots + 220(1.06)^{20} + 220(1.06)^{21}$

$= \dfrac{220(1.06)(1.06^{21} - 1)}{1.06 - 1}$

$= \$9326.30$

c. $y = 1 - \sin 2x$ for $0 \leq x \leq 2\pi$



d. x, x^2 and $5x$

i. If x, x^2 and $5x$ are part of an AP, then

$T_2 - T_1 = T_3 - T_2$

Where

$T_2 - T_1 = x^2 - x$ and

$T_3 - T_2 = 5x - x^2$

$\therefore\ x^2 - x = 5x - x^2$

$\therefore 2x^2 - 6x = 0$

ii. $2x^2 - 6x = 0$

$2x(x - 3) = 0$

$x = 0, 3$

Since $d = T_2 - T_1$, then when $x = 0$

$d = x^2 - x = 0$ (not possible)

When $x = 3$

$d = x^2 - x = 9 - 3 = 6$

(Check with $5x - x^2$ which $= 6$)

$\therefore$ Common difference $= 6$

e.

i.

$r^2 + (h - 3)^2 = 3^2$

$r^2 = 9 - (h - 3)^2$

$r^2 = 9 - h^2 + 6h - 9$

$r^2 = 6h - h^2$

$Sub\ r^2 = 6h - h^2$ into $V = \frac{1}{3}\pi r^2 h$

$V = \dfrac{1}{3}\pi(6h - h^2)h$

$\therefore V = \dfrac{\pi}{3}(6h^2 - h^3)$

ii.

$V = \dfrac{\pi}{3}\ (6h^2 - h^3)$

$V = 2\pi h^2 - \dfrac{\pi h^3}{3}$

$\dfrac{dV}{dh} = 4\pi h - \pi h^2$

$\dfrac{d^2V}{dh^2} = 4\pi - 2\pi h$

For stationary points, $\frac{dV}{dh} = 0$

$\therefore 4\pi h - \pi h^2 = 0$

$\pi h(4 - h) = 0$

$h = 0\ or\ 4$

$h \neq 0$ (as distance cannot be negative)

When $h = 4$

$\dfrac{d^2V}{dh^2} = 4\pi - 8\pi = -4\pi < 0, max$

$\therefore$ Height of cone is 4 cm