HSC Trial
EXAMINATION
2014

Student Number

Class (Please circle)

11M1   12M3   12M4   12M5   12M6

Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 16

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. Fully simplify the algebraic fraction: $\frac{x^3 - 8}{x^2 - 4}$.

(A) $\frac{x^2 - 2x + 4}{x - 2}$

(B) $x + 2$

(C) $\frac{x^2 + 4x + 4}{x + 2}$

(D) $\frac{x^2 + 2x + 4}{x + 2}$

2. The quadratic function $3x^2 - 5x + 2$ has roots α and β.

Which of the following statements is true?

(A) $2\alpha\beta = -\frac{4}{3}$

(B) $\alpha^2 + \beta^2 = \frac{13}{9}$

(C) $2\alpha + 3\beta = \frac{25}{3}$

(D) $\alpha^2\beta^2 = \frac{2}{9}$

3. Consider the series $S = 28 + 7 + \frac{7}{4} + \ldots$.

Find the difference between S_5 and S_3.

(A) $\frac{105}{64}$

(B) $\frac{231}{4}$

(C) $\frac{35}{64}$

(D) $\frac{231}{64}$

4. The point A has coordinates $(2, 7)$ and B has coordinates $(-2, 9)$.

What are the coordinates of the midpoint of the interval AB?

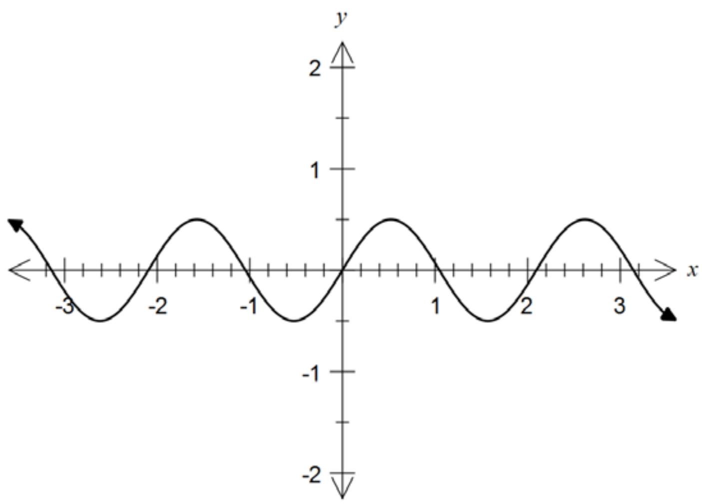
(A) $(0, 8)$

(B) $(-2, 1)$

(C) $(2, -1)$

(D) $(0, 3\frac{1}{2})$

5. What function would describe the graph shown?



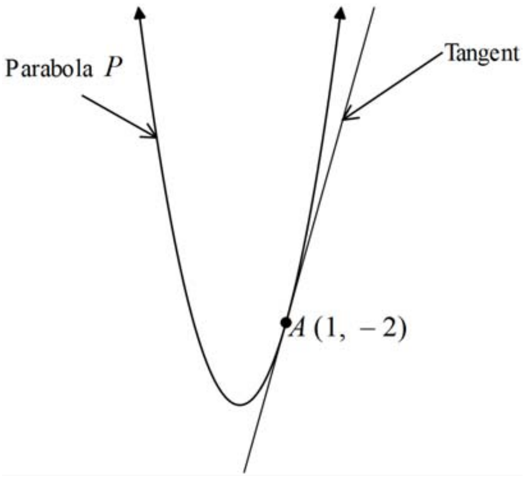
(A) $y = \frac{1}{2}\cos 3x$

(B) $y = \frac{1}{2}\sin 3x$

(C) $y = \frac{1}{2}\tan 3x$

(D) $y = \frac{1}{3}\sin 2x$

6. The diagram shows the parabola P and the tangent at the point $A(1, -2)$.



Which of the following equations might represent the normal to the parabola at the point A?

(A) $x - 3y + 5 = 0$

(B) $2x - 3y + 1 = 0$

(C) $x + 3y + 5 = 0$

(D) $x + 3y - 5 = 0$

7. For what domain and range is the function $y = \dfrac{1}{\sqrt{x-4}}$ defined?

(A) Domain: $x \geq 4$, Range: $y > 0$.

(B) Domain: $x > 4$, Range: $y > 0$.

(C) Domain: all real x, Range: all real y.

(D) Domain: $x < -2 \ \ x > 2$, Range: $y < 0$.

8. Which expression is a primitive function of $(4x-1)^3$?

(A) $12(4x-1)^2 + C$

(B) $\dfrac{1}{16}(4x-1)^3 + C$

(C) $\dfrac{1}{4}(4x-1)^4 + C$

(D) $\dfrac{1}{16}(4x-1)^4 + C$

9. What is the derivative of $(3x^2+1)^4$?

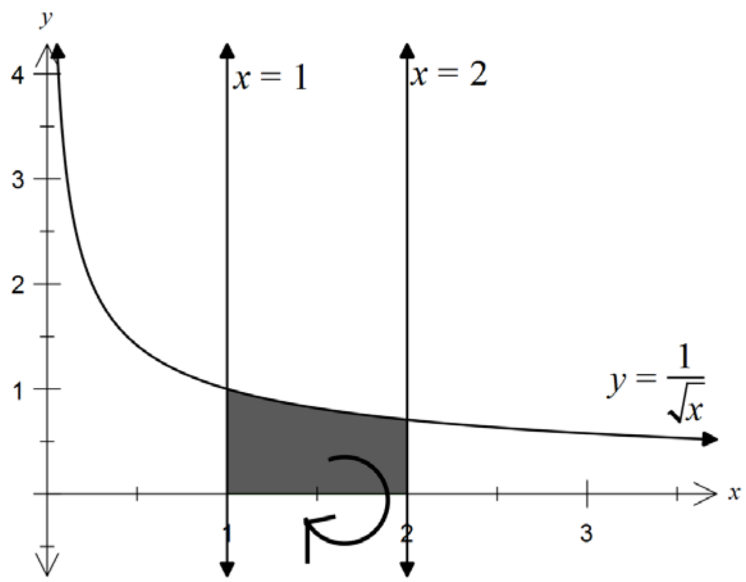
(A) $4(6x)^3$

(B) $6x(3x^2+1)^3$

(C) $24x(3x^2+1)^4$

(D) $24x(3x^2+1)^3$

10. The region between the functions $y=\frac{1}{\sqrt{x}}$, $x=1$ and $x=2$ is rotated about the x-axis. Find the volume of the solid formed.



(A) $\pi \ln 2$ cubic units

(B) $\ln 2$ cubic units

(C) $2(\sqrt{2}-1)$ cubic units

(D) $\ln \pi$ cubic units

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a new writing booklet.

(a) Evaluate e^3 correct to 3 significant figures. **1**

(b) Solve these simultaneous equations: **2**

$$2x - y = -1$$
$$5x + 3y = 25$$

(c) Differentiate the following functions:

(i) $x^3 - 4x^2 + 2$ **1**

(ii) $2x \cos 3x$ **2**

(d) If $f'(x) = 6x^2 + 5x - 1$ and $f(-1) = 5$, find an expression for $f(x)$. **2**

(e)

(i) Write the first three terms of the series whose general term is given by $T_n = \dfrac{2^n}{3^{n-1}}$. **1**

(ii) Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$. **2**

(f) Consider the quadratic equation $3x^2 + kx + 5 = 0$:

(i) For what values of k does the equation have no real roots? **3**

(ii) Describe the graph of $y = 3x^2 + kx + 5$ when k takes the values found in part (i). **1**

End of Question 11

Question 12 (15 marks) Use a separate writing booklet.

(a) A town called Benora is 15 kilometres away, on a bearing of 065° from another town called Andora. A third town, Calora is 42 kilometres East of Andora.

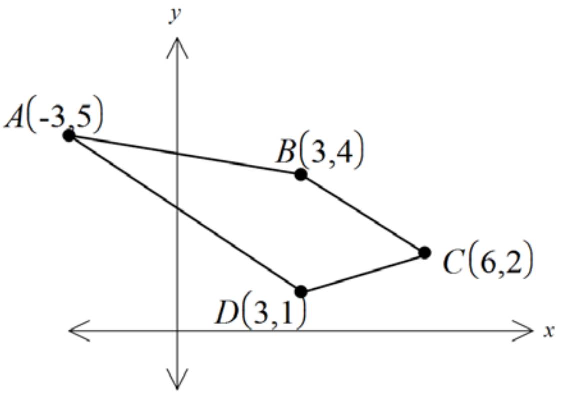
(i) Draw a diagram showing this information. **1**

(ii) Show that the distance from Benora to Calora is 29 kilometres, correct to the nearest kilometre. **2**

(iii) Find the bearing of Benora from Calora, correct to the nearest degree. **2**

(b) The points *A*(-3, 5), *B*(3, 4), *C*(6, 2) and *D*(3, 1) are the vertices of quadrilateral *ABCD*.

NOT TO SCALE



(i) Show that the equation of the line passing through *B* and *C* is $2x + 3y - 18 = 0$. **2**

(ii) Show that $AD \parallel BC$. **1**

(iii) Show that the perpendicular distance from point *D* to the line passing through *B* and *C* is $\frac{9\sqrt{13}}{13}$ units. **2**

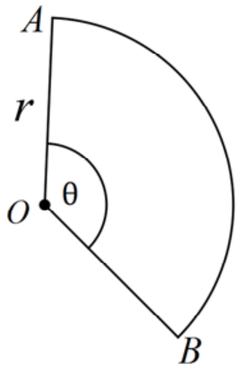
(iv) Show that *ABCD* is a trapezium. **1**

(v) Given that the distance between *B* and *C* is $\sqrt{13}$ units, calculate the exact area of quadrilateral *ABCD*. **2**

Question 12 continues on page 9

Question 12 (continued)

(c) Tim was told that sector *OAB* has an area of $\frac{25\pi}{6}$ square units. The arc *AB* is $\frac{5\pi}{3}$ units long.



Tim was asked to find the exact values of *r* and θ.

His working out is shown below:

$l = r\theta, \quad A = \frac{1}{2}r^2\theta$

$\therefore \quad \frac{1}{2}r^2\theta = \frac{25\pi}{6} \qquad (1)$

$r\theta = \frac{5\pi}{3} \qquad (2)$

$\frac{1}{2}r = \frac{5}{2} \qquad (3)$

$\therefore \quad r = 5$ units

(i) What operation did Tim perform on equations (1) and (2) to get to equation (3)? **1**

(ii) What is the value of θ? **1**

End of Question 12

Question 13 (15 marks) Use a separate writing booklet.

(a) Robyn and Maria start jobs at the beginning of the same year. Robyn's salary is higher than Maria's. Both Robyn's and Maria's employers pay into their superannuation funds at the beginning of each month.

Robyn's employer deposits \$550 per month into her superannuation fund which earns interest at 0.5% per month. Maria's employer deposits \$520 per month into her superannuation fund which earns 0.6% per month.

(i) Show that the amount of interest that Robyn's superannuation earned in the first year was \$218.48. **3**

(ii) Let A_n represent the amount after *n* months. Show that the amount in Robyn's superannuation fund after *n* months is given by: **1**

$$A_n = 110550\left(1.005^n - 1\right)$$

(iii) After how many months will the amount in Maria's superannuation fund be greater than the amount in Robyn's? **3**

(b) *Temp4U* is an employment agency which specialises in contracting temporary employees. They have analysed the number of job applications received over the last five years. They found that the demand (D), measured in hundreds, for temporary employment at time (*t* years) is given by the function:

$$D(t) = 4\sin\left(\frac{\pi}{4}t\right) + 7$$

(i) Find all the times in the next 12 years where demand will be at its peak. **3**

(ii) State the amplitude and period of *D(t)* and sketch its graph for the first twelve years. **3**

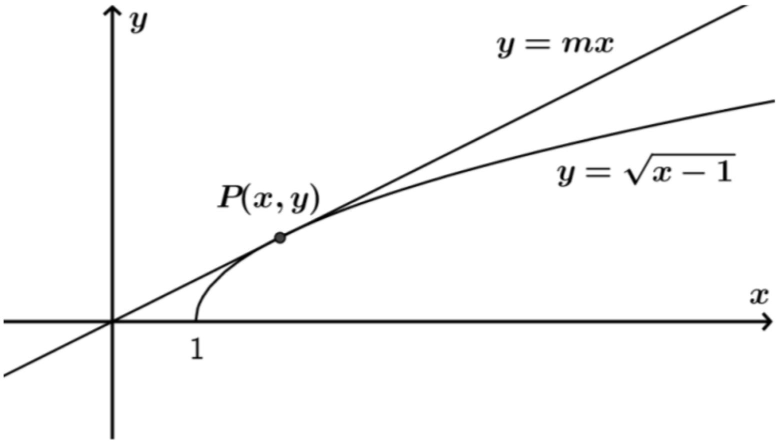
(c) Evaluate: $\lim_{x \to \infty} \frac{x^4 + 3x^2 + 2}{5x^4 + 1}$. **2**

End of Question 13

Question 14 (15 marks) Use a separate writing booklet.

(a) Use Simpson's rule to approximate $y = \int_1^5 \frac{dx}{x^2 + 1}$, using 5 function values. **3**

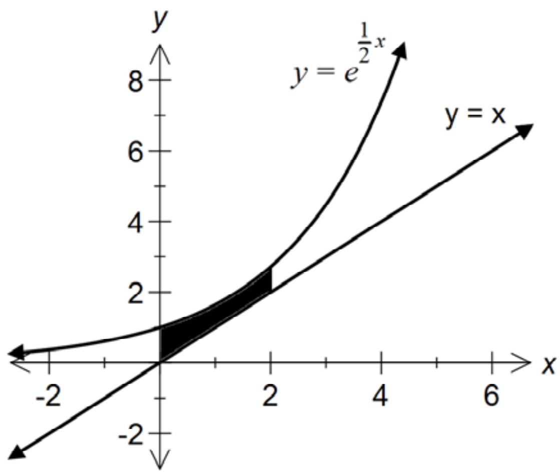
(b) The diagram shows the tangent $y = mx$ to the curve $y = \sqrt{x-1}$ at the point $P(x, y)$.



(i) Find the possible value(s) of m. **3**

(ii) Find the coordinates of the point $P(x, y)$. **2**

(c) The diagram shows the graphs of the functions $y = e^{\frac{1}{2}x}$ and $y = x$. The region between these 2 functions and the bounds $x = 0$ and $x = 2$ has been shaded. **3**



Calculate the exact area of the shaded region.

Question 14 continues on page 12

Question 14 (continued)

(d) For the parabola with equation $16y = x^2 - 4x - 12$:

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

End of Question 14

Question 15 (15 marks) Use a separate writing booklet.

(a)

(i) At which points on the curve $f(x)=\frac{x^3}{8}+1$ can a normal be drawn with a gradient of $-\frac{2}{3}$? **2**

(ii) At which point on the curve $f(x)$ will the normal be vertical? **1**

(b) The function $f(x) = xe^{-2x} + 1$ has first derivative $f'(x) = e^{-2x} - 2xe^{-2x}$ and second derivative $f''(x) = 4xe^{-2x} - 4e^{-2x}$.

(i) Find the value of x for which $f(x)$ has a stationary point. **1**

(ii) Find the values of x for which $f(x)$ is increasing. **1**

(iii) Find the value of x for which $f(x)$ has a point of inflection and determine where the graph $y = f(x)$ is concave upwards. **2**

(iv) Sketch the curve $y = f(x)$ for $-\frac{1}{2} \leq x \leq 4$. **2**

(v) Describe the behaviour of the graph for very large positive values of x. **1**

(c) Solve the equation $4\cos^2\theta = 6\sin\theta + 6$ in the domain $0 \leq \theta \leq 2\pi$. **3**

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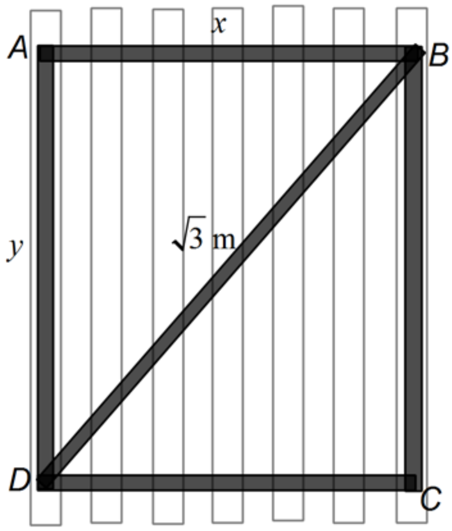
(d) Show that $\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$ for all integers $n \geq 1$. **2**

End of Question 15

Question 16 (15 marks) Use a separate writing booklet.

(a) A swinging gate is to be constructed from timber palings. It will require a support frame using 5 pieces of timber: *AB*, *AD*, *BD*, *BC* and *CD*.

$AB \| CD$ and $AD \| BC$. $AB = CD = x$ metres. $AD = BC = y$ metres. BD is $\sqrt{3}$ metres long.



(i) Find an expression for y in terms of x. **1**

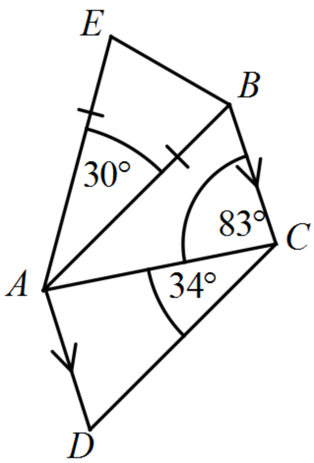
(ii) Show that the total length (L) of the timber pieces in the support frame is represented by $\boldsymbol{L} = 2\left(x + \sqrt{3 - x^2} + \dfrac{\sqrt{3}}{2}\right)$. **1**

(iii) The gate will have its maximum strength when the length of its support frame is maximised. For what value of x will the gate have maximum strength? **4**

Question 16 continues on page 15

Question 16 (continued)

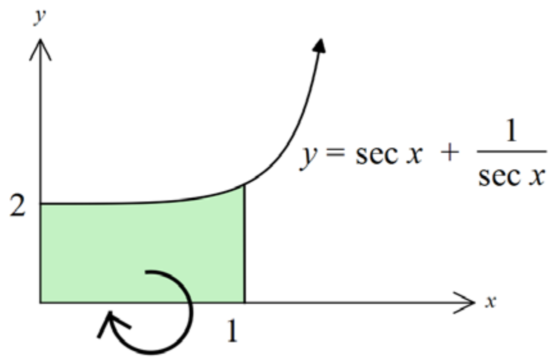
(b) In the diagram below: $AD \| BC$, $AE = AB$, $\angle BAE = 30^\circ$, $\angle BCA = 83^\circ$, $\angle ACD = 34^\circ$, $\angle EBC = 138^\circ$.



(i) Prove that $AB \| DC$. **2**

(ii) Prove that $\Delta ABC \equiv \Delta ACD$. **3**

(c) The area bounded by the function $y = \sec x + \dfrac{1}{\sec x}$, the y-axis and the line $x = 1$ is rotated about the x-axis.



(i) Show that $\left(\sec x + \dfrac{1}{\sec x}\right)^2 = \sec^2 x + \cos^2 x + 2$. **1**

(ii) Find the volume of the solid formed, given that: $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$, correct to 1 decimal place. **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

NORMANHURST BOYS HIGH SCHOOL

2014
TRIAL HSC
EXAMINATION

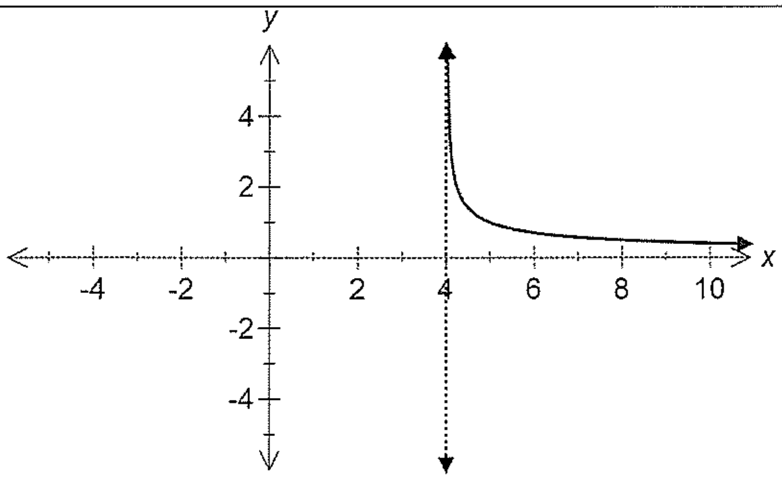
Mathematics

SOLUTIONS

Multiple Choice Worked Solutions

No	Working	Answer
1	$\frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$ (with $(x-2)$ cancelled) $= \frac{x^2+2x+4}{x+2}$	**D**
2	For the function $3x^2 - 5x + 2$, $a = 3, b = -5, c = 2$ $\alpha + b = -\frac{b}{a} = \frac{5}{3}$ $\alpha\beta = \frac{c}{a} = \frac{2}{3}$ $\therefore \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= \frac{25}{9} - \frac{4}{3}$ $= \frac{13}{9}$	**B**
3	The series is geometric with $a = 28$ *and* $r = \frac{1}{4}$. We can find the difference between S_5 and S_3 by calculating $S_5 - S_3$ or by working out $T_4 + T_5$. I will show the first method: $S_n = \frac{a(1-r^n)}{1-r}$ $S_5 = \frac{28\left(1-\left(\frac{1}{4}\right)^5\right)}{\frac{3}{4}}$ $= 37\ \frac{19}{64}$ $S_3 = \frac{28\left(1-\left(\frac{1}{4}\right)^3\right)}{\frac{3}{4}}$ $= 36\ \frac{3}{4}$ $S_5 - S_3 = \frac{35}{64}$	**C**
4	$M = \left(\frac{2 + -2}{2}, \frac{7+9}{2}\right)$ $= (0, 8)$	**A**
5	On inspection: the graph takes the shape of a sine or cosine function, passes through (0, 0) so can't be the cosine function offered,	**B**

	has amplitude = $\frac{1}{2}$ and frequency of $\frac{2\pi}{3}$. Therefore it is a sine function with $a = \frac{1}{2}, n = 3$ which is $y = \frac{1}{2}\sin 3x$.	
6	The normal is perpendicular to the tangent. Since the tangent has a positive slope, then the normal must have a negative slope. It must also pass through the point (1, -2). Options C and D have negative gradients (this can be seen quickly by rearranging into gradient-intercept form). Then substitute (1, -2) into equations C and D to see which of these lines passes through that point and therefore could be the equation of the normal. For C: $x + 3y + 5 = 0$ $1 + 3(-2) + 5 = 1 - 6 + 5$ $= 0$ For D: $x + 3y - 5$ $1 + 3(-2) - 5 = 1 - 6 - 5$ $= -10$ Therefore, option C is the only equation which might be the equation of the normal.	**C**
7	For the function $y = \frac{1}{\sqrt{x-4}}$, $\sqrt{x-4} \neq 0$ because the denominator cannot be zero. Therefore $x - 4 \neq 0$ $x \neq 4$ Also, we cannot find a real solution for the square root of a negative number, $x - 4 > 0$ so $x > 4$ Therefore the domain is $x > 4$. If we examine $\lim_{x \to \infty}$ for this function, we see that it approaches zero. Therefore, the range is $y > 0$. This can also be shown algebraically by rearranging the formula so that x is the subject: $x = \frac{1 + 4y^2}{y^2}$.	**B**

		
8		**D**
9	$\frac{d}{dx}\left(\left(3x^2+1\right)^4\right) = 4\left(3x^2+1\right)^3 (6x)$ $= 24x\left(3x^2+1\right)^3$	**D**
10	$V = \pi\int_a^b y^2\,dx$ $V = \pi\int_1^2 \frac{1}{x}\,dx$ $= \pi\left[\ln x\right]_1^2$ $= \pi(\ln 2 - \ln 1)$ $= \pi \ln 2$ cubic units	**A**

Trial HSC Examination 2014
Mathematics Course

Name ____________________ Teacher ________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

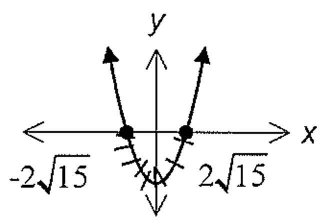
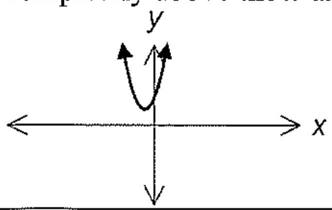
A ● B ⊗ C ○ D ○

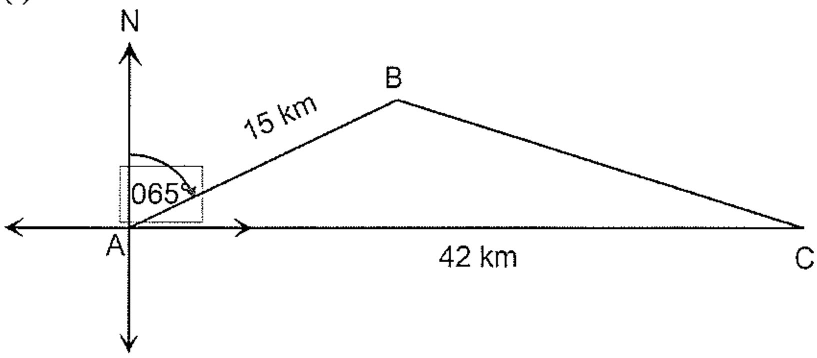
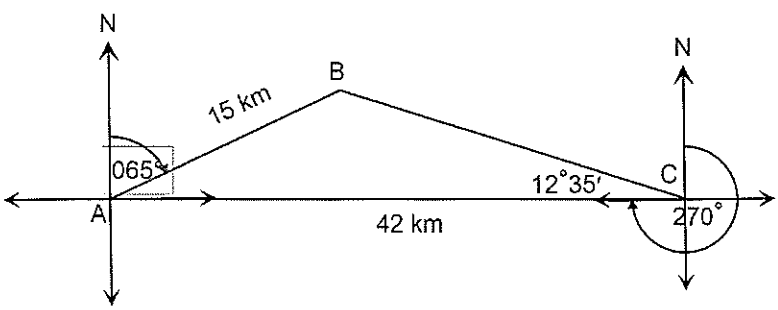
If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

	A	B	C	D
1.	○	○	○	●
2.	○	●	○	○
3.	○	○	●	○
4.	●	○	○	○
5.	○	●	○	○
6.	○	○	●	○
7.	○	●	○	○
8.	○	○	○	●
9.	○	○	○	●
10.	●	○	○	○

Question 11		2014	
	Solution	Marks	Allocation of marks
(a)	From calculator: $e^3 = 20.08553692$ ≈ 20.1	1	1 for correct rounding
(b)	$2x - y = -1$ ① $5x + 3y = 25$ ② $6x - 3y = -3$ ③ ((1) × 3) $11x = 22$ ④ ((2) + (3)) $\therefore x = 2$ ⑤ $2(2) - y = -1$ *substituting ⑤ into ①* $4 - y = -1$ $-y = -5$ $\therefore y = 5$	1 1	For mostly correct. Second mark if two correct solutions with logical working. (Could use substitution method or graphical solution also)
(c)	(i) $\frac{d}{dx}\left(x^3 - 4x^2 + 2\right) = 3x^2 - 8x$	1	1 for correct answer
	(ii) let $u = 2x$ $v = \cos 3x$ $u' = 2$ $v' = -3\sin 3x$ $\frac{d}{dx}(2x\cos 3x) = vu' + uv'$ $= \cos 3x \times 2 + 2x \times (-3\sin 3x)$ $= 2\cos 3x - 6x\sin 3x$	1 1	1 mark if correct progress made using product rule 2 marks for correct answer
(d)	$f(x) = \int 6x^2 + 5x - 1 \; dx$ $= 2x^3 + \frac{5}{2}x^2 - x + C$ when $x = -1$, $f(x) = 5$ $\therefore 2(-1)^3 + \frac{5}{2}(-1)^2 - (-1) + C = 5$ $-2 + \frac{5}{2} + 1 + C = 5$ $\frac{3}{2} + C = 5$ $C = \frac{7}{2}$ $\therefore f(x) = 2x^3 + \frac{5}{2}x^2 - x + \frac{7}{2}$	1 1	For integration. For evaluating the constant.

Question 11		2014	
	Solution	Marks	Allocation of marks
(e)	(i) $T_1 = 2$ $T_2 = \frac{4}{3}$ $T_3 = \frac{8}{9}$ $\therefore 2, \frac{4}{3}, \frac{8}{9}, \ldots\ldots\ldots$	1	For correct 3 terms.
	(ii) $a = 2$ $\frac{T_2}{T_1} = \frac{T_3}{T_2} = r$ $\frac{4}{3} \times \frac{1}{2} = \frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$	1	.for common ratio
	$\therefore r = \frac{2}{3}$ $\qquad S_\infty = \frac{2}{1 - \frac{2}{3}} = 6$	1	For correct solution
(f)	(i) Function has no real roots when the discriminant is less than zero. $\Delta < 0$ $b^2 - 4ac < 0$ $k^2 - 4(3)(5) < 0$	1	Letting $\Delta = 0$.
	Let $k^2 = 60$ $k = \pm\sqrt{60}$ $= \pm 2\sqrt{15}$ $k^2 < 2\sqrt{15}$ in the identified region below the x-axis $\therefore -2\sqrt{15} < k < 2\sqrt{15}$	1	Solving $k^2 = 60$.
		1	Testing for region.
	(ii) When a function has no real roots, it does not touch the x-axis. Since the coefficient of x^2 is positive, this parabola will be completely above the x-axis. 	1	For valid explanation or diagram showing that it is not touching x-axis, mentioning or showing a positive definite parabola.

Question 12		**2014**	
	Solution	Marks	Allocation of marks
(a)	(i) 	1	
	(ii) $\angle CAB = 90 - 65$ $= 25^\circ$ $BC^2 = 15^2 + 42^2 - 2(15)(42)\cos 25^\circ$ $= 847.0521883$	1	For correct substitution Into cosine rule
	$BC = \sqrt{847.0521883}$ ≈ 29	1	For correct verification
	(iii) $\dfrac{\sin \angle BCA}{15} = \dfrac{\sin 25}{29.10416101}$ $\sin \angle BCA = 15 \times \dfrac{\sin 25}{29.10416101}$ $\angle BCA = \sin^{-1}\left(15 \times \dfrac{\sin 25}{29.10416101}\right)$ $= 12^\circ 35'$ OR $12^\circ 29'$ using BC=29	1	Finding $\angle BCA$.
	 Bearing = 270° + 12°35' ≈ 283°	1	Correct bearing.

Question 12		2014	
	Solution	Marks	Allocation of marks
(b)	$A(-3, 5)$, $B(3, 4)$, $C(6, 2)$ and $D(3, 1)$ (i) For the line passing through B and C: $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{2-4}{6-3}$ $= -\frac{2}{3}$ $y - y_1 = -\frac{2}{3}(x - x_1)$ $y - 4 = -\frac{2}{3}(x-3)$ $3y - 12 = -2x + 6$ $2x + 3y - 18 = 0$ (ii) For AD||BC, must have same gradients. We already know gradient of BC from (i). $m_{AD} = \frac{1-5}{3+3}$ $= -\frac{4}{6}$ $= -\frac{2}{3}$ $= m_{BC}$	1 1 1	Various methods may be used. 1 mark if valid approach with small error, not leading to final solution. Second mark for reaching solution. OR 1 mark each for substitution into equation to verify

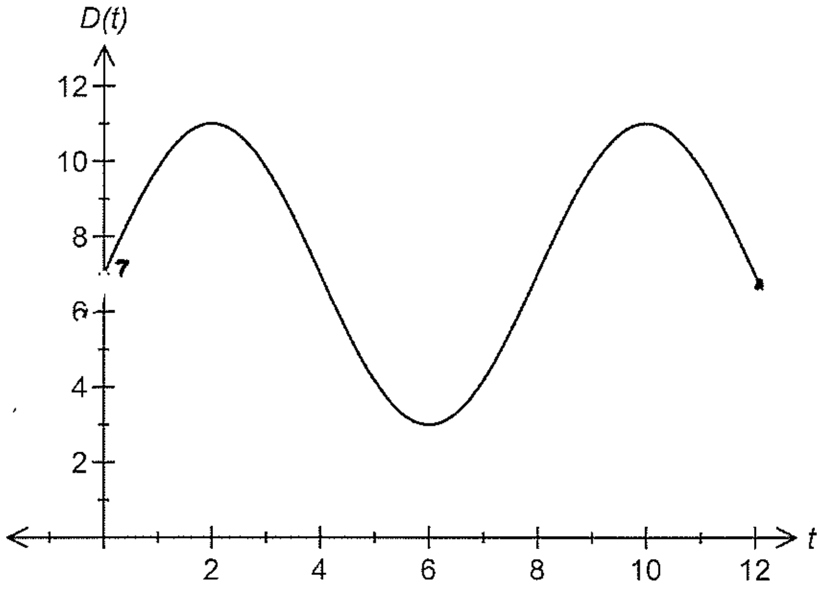
Question 12		2014	
	Solution	Marks	Allocation of marks
(b)	(iii) $2x + 3y - 18 = 0, \quad (3, 1)$ $d = \dfrac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2 + b^2}}$ $= \dfrac{\lvert 2(3) + 3(1) - 18 \rvert}{\sqrt{2^2 + 3^2}}$	1	Correct substitution.
	$= \dfrac{\lvert -9 \rvert}{\sqrt{13}}$ $= \dfrac{9}{\sqrt{13}} \times \dfrac{\sqrt{13}}{\sqrt{13}}$	1	Rationalising denominator.
	$= \dfrac{9\sqrt{13}}{13}$ *units*		
	(iv) In ABCD: AD||BC (shown above) If AB||CD then we have a parallelogram. Show that AB is not || to CD. ∴ ABCD is a trapezium (one pair of opposite sides parallel)	1	Valid reasoning. Could show that the lengths of AD and BC are not equal or gradients not equal instead.
	(v) For the area of a trapezium, we need to know the lengths of the parallel sides and perpendicular height. We must calculate the length of AD. $d^2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\text{AD} = \sqrt{(3 + 3)^2 + (1 - 5)^2}$ $= \sqrt{36 + 16}$ $= \sqrt{52}$	1	Finding length AD.
	$= 2\sqrt{13}$ *units* $A = \dfrac{1}{2}h(a + b)$ $= \dfrac{1}{2} \cdot \dfrac{9\sqrt{13}}{13}(\sqrt{13} + 2\sqrt{13})$ $= \dfrac{9\sqrt{13}}{26}(3\sqrt{13})$ $= \dfrac{27 \times 13}{26}$ $= 13\frac{1}{2}$ *square units*	1	Correct area.

Question 12		2014	
	Solution	Marks	Allocation of marks
(c)	(i) Tim has divided equation (1) by equation (2)	1	
	(ii) $r\theta = \frac{5\pi}{3}$		
	$5\theta = \frac{5\pi}{3}$		
	$\theta = \frac{\pi}{3}$	1	Correct value for θ.

Question 13		2014	
	Solution	Marks	Allocation of marks
(a)	(i) $A_1 = 550(1.005)^{12}$		
	$A_2 = 550(1.005)^{11}$		
	$A_{12} = 550(1.005)$		
	Total after 12 months		
	$= 550\left(1.005 + 1.005^2 + \ldots + 1.005^{12}\right)$	1	Constructing series.
	$= 550\left(\frac{1.005\left(1.005^{12} - 1\right)}{0.005}\right)$		
	$\approx \$6818.48$	1	Evaluating A_{12}.
	Interest $= 6818.48 - (12 \times 550)$	1	Subtracting employer contributions.
	$= \$218.48$		

Question [redacted] 13		2014	
	Solution	Marks	Allocation of marks
	(ii)		
	$A_n = 550\dfrac{1.005(1.005^n - 1)}{0.005}$	1	
	$= 110550(1.005^n - 1)$		
	(iii) For Maria:		
	$A_n = 520\left(\dfrac{1.006(1.006^n - 1)}{0.006}\right)$		
	$= \dfrac{261560}{3}(1.006^n - 1)$	1	Evaluating Maria's A_n.
	Let A_R represent A_n for Robyn and A_M represent A_n for Maria. We want $A_M > A_R$:		
	$\dfrac{261560}{3}(1.006^n - 1) > 110550(1.005^n - 1)$	1	Setting up inequality.
	$\dfrac{1.006^n - 1}{1.005^n - 1} > 110550 \div \dfrac{261560}{3}$		
	$\dfrac{1.006^n - 1}{1.005^n - 1} > 1.267969108$		
	By trial and error:		
	when $n = 102$ LHS = 1.267757392		
	when $n = 103$ LHS = 1.268504921		
	∴ *Maria's fund has more money in it than Robyn's after 103 months.*	1	Correct answer.

Question [redacted]13		**2014**	
	Solution	Marks	Allocation of marks
(b)	(i) Maximum will occur when $D'(t) = 0$ and $D''(t) < 0$.		
	$D'(t) = 4 . \frac{\pi}{4}\cos\left(\frac{\pi}{4}t\right)$		
	$= \pi\cos\left(\frac{\pi}{4}t\right)$	1	Correct expression for $D'(t)$.
	$Let\ D'(t) = 0$		
	$\pi\cos\left(\frac{\pi}{4}t\right) = 0$		
	$\cos\left(\frac{\pi}{4}t\right) = 0$		
	$\frac{\pi}{4}t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$		
	$t = 2,\ 6,\ 10,\ 14, \ldots$	1	Evaluating t.
	Check concavity		
	$D''(t) = -\frac{\pi^2}{4}\sin\left(\frac{\pi}{4}t\right)$		
	$when\ t = 2,\ D''(t) = -\frac{\pi^2}{4}\sin\left(\frac{\pi}{2}\right)$		
	< 0		
	$\therefore max\ at\ t = 2$		
	$when\ t = 6,\ D''(t) = -\frac{\pi^2}{4}\sin\left(\frac{3\pi}{2}\right)$		
	> 0	1	Testing for nature of turning points leading to correct values for t.
	$\therefore min\ at\ t = 6$		
	Due to the nature of the sine curve, we know that the next turning point will be a maximum. Therefore maximum demand will occur at 2 years and 10 years.		
	OR		
	D(t) is a maximum when $\sin(\frac{\pi}{4}t) = 1$	1	
	$\therefore \frac{\pi}{4}t = \frac{\pi}{2}, \frac{5\pi}{2}$	1	
	$\therefore t = 2,10$	1	

Question 13		2014	
	Solution	Marks	Allocation of marks
	(ii) $a = 4,$ $period = \frac{2\pi}{\frac{\pi}{4}}$ $= 8$	1	Amplitude <u>and</u> period.
		1 1	Turning points correct. Correct y-intercept, $t \geq 0$ Deduct 1 for lack of consistent scale and/or poor shape of curve.
(c)	$\lim_{x \to \infty} \frac{x^4 + 3x^2 + 2}{5x^4 + 1} = \lim_{x \to \infty} \frac{1 + \frac{3}{x^2} + \frac{2}{x^4}}{5 + \frac{1}{x^4}}$	1	Dividing by highest power of x.
	$= \frac{1 + 0 + 0}{5 + 0}$		
	$= \frac{1}{5}$	1	Correct answer.

Question 14		2014	
	Solution	Marks	Allocation of marks
(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$\frac{1}{x^2+1}$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{5}$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{17}$</td><td>$\frac{1}{26}$</td></tr></table>	1	Table of values
	$A = \frac{h}{3}(y_0 + y_5 + 4(y_1 + y_3) + 2y_2)$		
	$= \frac{1}{3}\left(\frac{1}{2} + \frac{1}{26} + 4\left(\frac{1}{5} + \frac{1}{17}\right) + 2\left(\frac{1}{10}\right)\right)$	1	Correct substitution.
	$= \frac{392}{663}$ *square units* (= 0.5913 to 4 dp)	1	Correct answer.
(b)	(i) $mx = \sqrt{x-1}$ $m^2x^2 = x-1$ $m^2x^2 - x + 1 = 0$	1	Forms quadratic equation
	$\Delta = 0$ $1 - 4m^2 = 0$ $(1-2m)(1+2m) = 0$		Correct derivative with no further work, award 0
	$m = \frac{1}{2}$ $m = -\frac{1}{2}$ (reject since m > 0)	1	Forms discriminant and solves for m
	$\therefore m = \frac{1}{2}$	1	Gives correct value for m
	(ii) substitute $m = \frac{1}{2}$ into $m^2x^2 - x + 1 = 0$		
	$\frac{1}{4}x^2 - x + 1 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$ $y = 1$	1	For sub. m= 0.5, or other correct method. (such as substituting the correct derivative from (i) into $mx = \sqrt{x+1}$)
	$\therefore P(2,1)$	1	Correct coordinates. .

Question 14		2014	
	Solution	Marks	Allocation of marks
(c)	$A = \int_0^2 \left(e^{\frac{1}{2}x} - x\right) dx$	1	Setting up difference of integrals.
	$= \left[2e^{\frac{1}{2}x} - \frac{x^2}{2}\right]_0^2$	1	Correct integration.
	$= \left(2e^{\frac{1}{2}(2)} - \frac{2^2}{2}\right) - \left(2e^{\frac{1}{2}(0)} - \frac{0^2}{2}\right)$		
	$= 2e - 2 - 2$		
	$= 2(e-2) units^2$	1	Correct answer.
(d)	(i) $16y = x^2 - 4x - 12$		
	$16y + 12 = (x-2)^2 - 4$		
	$16y + 16 = (x-2)^2$	1	Completing square.
	$16(y+1) = (x-2)^2$		
	Vertex has coordinates (2, -1).	1	Correct vertex.
	(ii) This parabola is in the form $(x-h)^2=4a(y-k)$ therefore it is concave up.		
	$4a = 16$		
	$\therefore\ a = 4$		
	So the focus is 4 units above the vertex and has coordinates	1	Correct focus.
	(2, 3).		
	(iii) So directrix is 4 units below vertex. $\therefore y = -1 - 4$	1	Equation of the directrix
	$y = -5$ *is the equation of the directrix*		

Question 15		2014	
	Solution	Marks	Allocation of marks
(a)	(i) $f(x)=\frac{x^3}{8}+1$ $f'(x)=\frac{3x^2}{8}$ $\frac{3x^2}{8}=\frac{3}{2}$ $6x^2=24$ $x^2=4$ $x=2 \quad or \quad x=-2$ $\quad\therefore$ Normals can be drawn at the points $(2,2)$ and $(-2,0)$ $y=2 \quad\quad y=0$	1 1	Finds correct derivative and relates it to the gradient of the normal Finds the correct points
	(ii) At the point $(0,1)$ the normal is vertical.	1	Correct point

Question 15		2014	
	Solutio	Marks	Allocation of marks

(b)

(i) Stationary point at $f'(x) = 0$

$$f'(x) = e^{-2x} - 2xe^{-2x} = 0$$
$$e^{-2x}(1-2x) = 0$$
$$1-2x = 0 \quad \left(e^{-2x} \neq 0\right)$$
$$x = \tfrac{1}{2}.$$

Marks: 1 — Correct value

(ii) $f(x)$ is increasing when $f'(x) > 0$.

$$f'(x) = e^{-2x}(1-2x) > 0$$
$$1-2x > 0 \quad \left(e^{-2x} > 0\right)$$
$$1 > 2x$$
$$x < \tfrac{1}{2}.$$

Marks: 1 — Correct value

(iii) $f(x)$ has a point of inflection if $f''(x) = 0$ and $f''(x)$ changes sign.

$$f''(x) = 4xe^{-2x} - 4e^{-2x} = 0$$
$$4e^{-2x}(x-1) = 0$$
$$x - 1 = 0 \quad \left(4e^{-2x} \neq 0\right)$$
$$x = 1.$$

Marks: 1 — Use of second derivative to find $x = 1$

$f''(x) = 4e^{-2x}(x-1)$.
$4e^{-2x}$ is always positive,
so when $x < 1$, $f''(x) < 0$
$x > 1$, $f''(x) > 0$.

OR

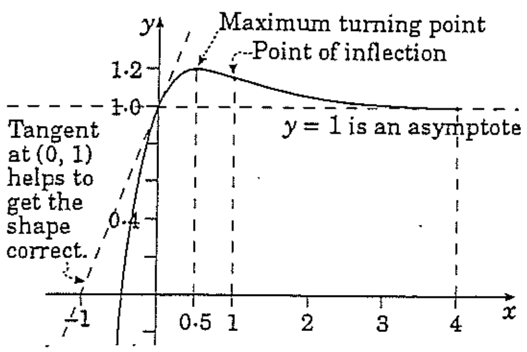
x	0	1	2
$f''(x)$	-4	0	0·07
	$-$		$+$

$\therefore$ Concavity changes from negative (concave down) to positive (concave up).
$\therefore$ Point of inflection occurs at $x = 1$, and graph is concave up for $x > 1$.

Marks: 1 — $x>1$ <u>with clear justification</u>

(iv)

x	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	4
$f(x)$	$-0{\cdot}359$	1	1·184	1·135	1·001
		$f'(0) = 1$	Stationary point, from (i)	Point of inflection, from (iii)	
	Increasing, from (ii)			Decreasing	
	Concave down				Concave up, from (iii)



Marks: 1 — Shows turning point + x & y intercepts

Marks: 1 — Show pt of inflexion and curve approaching $y=1$

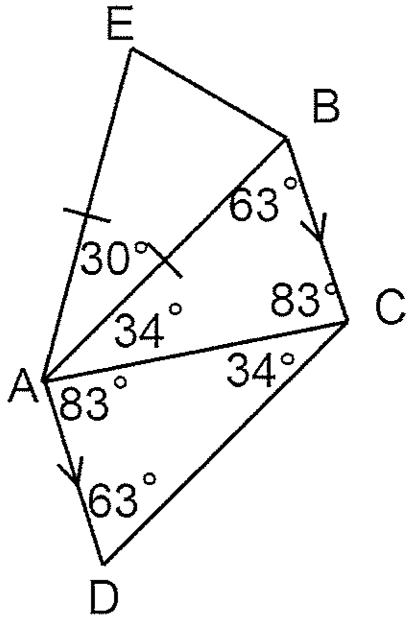
(v) As $x \to \infty$, $xe^{-2x} \to 0$,
$\therefore \quad f(x) = xe^{-2x} + 1 \to 1$.
$\therefore$ The graph approaches the line $y = 1$.

Marks: 1 — Correct answer

Question 15		2014	
	Solution	Marks	Allocation of marks
(c)	$4\cos^2\theta = 6\sin\theta + 6$ $4(1-\sin^2\theta) = 6\sin\theta + 6$ $4 - 4\sin^2\theta = 6\sin\theta + 6$ $4\sin^2\theta + 6\sin\theta + 2 = 0$ $\frac{(4\sin\theta + 4)(4\sin\theta + 2)}{4} = 0$ $4\sin\theta = -4$ *or* $4\sin\theta = -2$ $\sin\theta = -1$ $\quad\sin\theta = -\frac{1}{2}$ *sine is negative in the 3rd and 4th quadrants* $\therefore \quad \theta=\frac{3\pi}{2}$ *or* $\theta = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$ $= \frac{7\pi}{6},\ \frac{11\pi}{6}$	1 1 1	Using trig identity to get in terms of sine only. Factorisation producing values of sinθ Showing 3 solutions.
(d)	$\frac{1}{\sqrt{n} + \sqrt{n+1}}$ $= \frac{1}{\sqrt{n} + \sqrt{n+1}} \times \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} - \sqrt{n+1}}$ $= \frac{\sqrt{n} - \sqrt{n+1}}{n-(n+1)}$ $= \frac{\sqrt{n} - \sqrt{n+1}}{-1}$ $= \sqrt{n+1} - \sqrt{n}$	1 1	Multiplying by conjugate. Simplifying.

Question 16		2014	
	Solution	Marks	Allocation of marks
(a)	(i) $y = \sqrt{(\sqrt{3})^2 - x^2}$ $= \sqrt{3 - x^2}$ (ii) $L = 2y + 2x + \sqrt{3}$ $= 2\sqrt{3 - x^2} + 2x + \sqrt{3}$ $= 2\left(x + \sqrt{3 - x^2} + \frac{\sqrt{3}}{2}\right)$	1 1	
(a)	(iii) $L = 2x + 2(3 - x^2)^{\frac{1}{2}} + \sqrt{3}$ $L' = 2 + 2\left(\frac{1}{2}\right)(-2x)(3 - x^2)^{-\frac{1}{2}}$ $= 2 - \frac{2x}{\sqrt{3 - x^2}}$ *Turning points occur when $L' = 0$* $0 = 2 - \frac{2x}{\sqrt{3 - x^2}}$ $\frac{2x}{\sqrt{3 - x^2}} = 2$ $\frac{x}{\sqrt{3 - x^2}} = 1$ $x = \sqrt{3 - x^2}$ $x^2 = 3 - x^2$ $2x^2 = 3$ $x^2 = \frac{3}{2}$ $x = \pm\sqrt{\frac{3}{2}}$ *units* *we can ignore the negative answer since we are dealing with a length* $\therefore$ *there is a turning point at* $x = \sqrt{\frac{3}{2}}$	1 1	Differentiation. Solution for x.

Question 16		2014	
	Solution	Marks	Allocation of marks
(a)	(continued) We need to check if our turning point is a maximum as required. $L' = 2 - 2x(3-x^2)^{-\frac{1}{2}}$ let $u = -2x \quad v = (3-x^2)^{-\frac{1}{2}}$ $u' = -2 \quad v' = \left(-\frac{1}{2}\right)(-2x)(3-x^2)^{-\frac{3}{2}}$ $= x(3-x^2)^{-\frac{3}{2}}$ $L'' = \frac{d}{dx}(2) + \frac{d}{dx}\left(-2x(3-x^2)^{-\frac{1}{2}}\right)$ $= 0 + vu' + uv'$ $= -2(3-x^2)^{-\frac{1}{2}} - 2x^2(3-x^2)^{-\frac{3}{2}}$ $= -\frac{2}{\sqrt{3-x^2}} - \frac{2x^2}{\sqrt{(3-x^2)}^3}$ $= \frac{-2\left(\sqrt{3-x^2}\right)^2 - 2x^2}{\sqrt{(3-x^2)}^3}$ $= -\frac{6}{\sqrt{(3-x^2)}^3}$ when $x = \sqrt{\frac{3}{2}}$ $L'' \approx -3.27$ $\therefore$ the turning point at $x = \sqrt{\frac{3}{2}}$ is a maximum	1 1	Correct test for maximum using L'' OR L' If using L', values of L' should be shown either side of the turning point Correct conclusion

Question 16		**2014**	
	Solution	Marks	Allocation of marks
(b)	(i) 		
	In ΔABE : AE = EB (given) $\therefore \Delta ABE$ is isosceles $\therefore \angle AEB = \angle ABE = \frac{180-30}{2}$ (angle sum of Δ and equal base angles in isosceles Δ) $= 75°$ $\angle CBA = \angle EBC - \angle ABE$ $= 138 - 75$ $= 63^\circ$	1	For finding $\angle CBA$.
	$\angle ABC + \angle BCD = 63 + 83 + 34$ $= 180^\circ$ $\therefore$ AB||CD (cointerior angles are supplementary)	1	For valid reasoning.
	(ii) In ΔABC and ΔACD : AB || CD (above) $\angle BAC = \angle ACD = 34°$ (alternate angles in || lines)	1	
	AD || BC (given) $\therefore$ $\angle CAD = \angle BCA = 83°$(alternate angles on || lines)	1	
	Side AC is common $\therefore \Delta ABC \equiv \Delta ACD$ (AAS)	1	

Question 16		2014	
	Solution	Marks	Allocation of marks
(c)	(i) $\left(\sec x + \dfrac{1}{\sec x}\right)^2 = \sec^2 x + \dfrac{2\sec x}{\sec x} + \dfrac{1}{\sec^2 x}$	1	
	$= \sec^2 x + 2 + \cos^2 x$		
	(ii) $V = \pi \int y^2\, dx$		
	$= \pi \int_0^1 \left(\sec^2 x + \cos^2 x + 2\right) dx$		
	$V = \pi\left[\int_0^1 \sec^2 x\, dx + \dfrac{1}{2}\int_0^1 (\cos 2x + 1)\, dx + \int_0^1 2\right] dx$	1	For correct substitution into Volume formula
	$= \pi\left[\tan x + \dfrac{1}{2}\left(\dfrac{1}{2}\sin 2x + x\right) + 2x\right]_0^1$	1	Integration.
	$= \pi\left[\tan x + \dfrac{\sin 2x}{4} + \dfrac{x}{2} + 2x\right]_0^1$		
	$= \pi\left[\left(\tan 1 + \dfrac{\sin 2}{4} + \dfrac{1}{2} + 2\right) - \left(\tan 0 + \dfrac{\sin 0}{4} + 0 + 0\right)\right]$		
	$= \pi\left(\tan 1 + \dfrac{\sin 2}{4} + \dfrac{5}{2}\right)$		
	≈ 13.5 *cubic units*	1	Answer.