Equations Study Summary

How do you solve an equation?

We move all the numbers away from the letter until the letter is on its own.

E.g. $a = 4$

How do we move a number away from the letter?

By doing the opposite operation to both sides.

One Step Equations

$x + 3 = 10$
   -3   -3

$x = 7$

$y - 4 = 6$
   +4   +4

$y = 10$

$3m = 12$
$\div 3 \quad \div 3$

$m = 4$

$\dfrac{p}{5} = 7$
$\times 5 \quad \times 5$

$p = 35$

Two Step Equations

$3n - 6 = 12$

Which number do we move first?

$3n - 6 = 12$

| or | ?

When we solve equations, we work backwards.
This means we usually move + or - numbers,

then

we move x or ÷ numbers.

$3n - 6 = 12$
$ + 6 \quad + 6$

$3n = 18$
$\div 3 \quad \div 3$

$n = 6$

$$\frac{a}{3} + 7 = 11$$
$$\phantom{\frac{a}{3}} - 7 \quad - 7$$

$$\frac{a}{3} = 4$$
$$\times 3 \quad \times 3$$

$$a = 12$$

Two Step Equations Special Variation 1
(Long fraction line)

We usually move + or - numbers, before we move x or ÷ numbers.

$$\frac{k - 2}{5} = 4$$

In this case, we must move the

bottom number first.

$$\frac{k - 2}{\cancel{5}} = 4$$

$$\cancel{\times 5} \qquad \times 5$$

$$k \cancel{- 2} = 20$$

$$\cancel{+ 2} \qquad + 2$$

$$k = 22$$

Two Step Equations Special Variation 2
(Brackets)

Expand the brackets first.

$$5(m + 10) = 80$$

$$5(m + 10) = 80$$

$$5m + 50 = 80$$

$$5m \cancel{+ 50} = 80$$

$$\cancel{- 50} \quad - 50$$

$$\cancel{5}m = 30$$

$$\cancel{\div 5} \quad \div 5$$

$$m = 6$$

Variations Recap

Long fraction line?
Move bottom number first.

Brackets?
Expand brackets first.

Equations with letters on both sides

"Combine" the letter terms by
moving the smaller letter term first.

$5c + 10 = 3c + 30$

Which is the smaller letter term?　$3c$

What is the opposite of a $+ 3c$?

$- 3c$

$5c + 10 = \cancel{3c} + 30$
$- 3c \qquad \cancel{- 3c}$

$2c + \cancel{10} = 30$
$\qquad \cancel{- 10} \ - 10$

$\cancel{2}c \ = 20$
$\cancel{\div 2} \quad \div 2$

$c \ = \ 10$

$3y + 14 = 6y - 1$

Which is the smaller letter term?　$3y$

What is the opposite of a $+ 3y$?

$- 3y$

$\cancel{3y} + 14 = 6y - 1$
$\cancel{- 3y} \qquad - 3y$

$\qquad 14 = \ 3y - 1$

At this point, you may be concerned
that the combined letter term seems to
be on the "wrong" side.

$\cancel{3y} + 14 = 6y - 1$
$\cancel{- 3y} \qquad - 3y$

$\qquad 14 = \ 3y \cancel{- 1}$
$\qquad +1 \qquad \cancel{+ 1}$

$\qquad 15 = \cancel{3}y$
$\qquad \div 3 \ \cancel{\div 3}$

$\qquad 5 = y$

To "fix" this solution, we simply
swap sides!

$y = 5$ 　Done!

Solving Fraction Equations

$$\frac{a}{3} + \frac{3a}{4} = 2$$

Multiply every term by a "magic number"

that 3 and 4 both divide into.

Multiply every term by 12.

$$12^{4}\left(\frac{a}{3^{1}}\right) + 12^{3}\left(\frac{3a}{4^{1}}\right) = 12\left(2\right)$$

$$4a + 9a = 24$$

$$13a = 24$$

$$\div 13 \quad \div 13$$

$$a = \frac{24}{13}$$

$$\frac{a+5}{3} = \frac{a-1}{8}$$

$$^{8x}\frac{a+5}{3} = ^{3x}\frac{a-1}{8}$$

$$8(a+5) = 3(a-1)$$

$$8a + 40 = 3a - 3$$

Move the smaller letter term first.

$$8a + 40 = 3a - 3$$
$$- 3a \qquad - 3a$$

$$5a + 40 = -3$$
$$-40 \qquad - 40$$

$$5a = -43$$
$$\div 5 \qquad \div 5$$

$$a = \frac{-43}{5}$$

Checking Equation Solutions

We can check our solution to any equation.

To check a solution:

Replace the letter with your solution.

If this makes the Left Side = Right Side,

your solution is correct!

$5y - 6 = 9$ $y = 4$

$5(4) - 6 = 9$

$20 - 6 = 9$

$14 = 9$

Left Side ≠ Right Side

Your solution is NOT correct!

$3a + 4 = 19$ $a = 5$

$3(5) + 4 = 19$

$15 + 4 = 19$

$19 = 19$

Left Side = Right Side

Your solution is correct!

$3a + 2 = 5a - 12$ $a = 7$

$3(7) + 2 = 5(7) - 12$

$21 + 2 = 35 - 12$

$23 = 23$

Left Side = Right Side

Your solution is correct!

Solving Inequations

We solve inequations using the same methods as solving equations...
except

if you multiply or divide by a negative,

the direction of the inequality sign changes.

Solving Inequations

If you multiply or divide by a negative
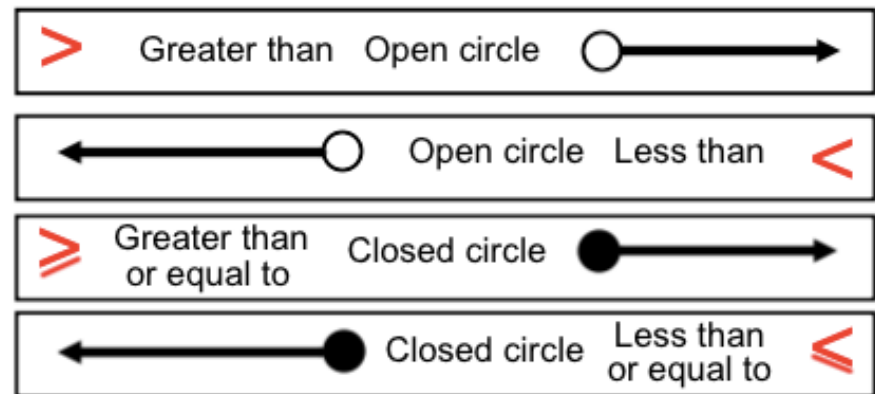the direction of the inequality sign changes.

$$3x > 12$$
$$\div 3 \quad \div 3$$
$$x > 4$$

$$\frac{m}{-5} \leqslant 2$$
$$x-5 \quad x-5$$
$$m \geqslant -10$$

$$^{+}7 - 4a > -17$$
$$-7 \qquad -7$$
$$- 4a > -24$$
$$\div -4 \quad \div -4$$
$$a < 6$$

Because we divided by a negative, we change the direction of the inequality.

Graphing inequalities on the number line

$>$ Greater than   Open circle

   Open circle   Less than $<$

$\geqslant$ Greater than or equal to   Closed circle

   Closed circle   Less than or equal to $\leqslant$

Graph your answer on the number line.

$x > 4$ x is greater than 4, but not equal to 4.

Open circle around 4.

Graph your answer on the number line.

$x \leqslant 7$ x can be __less__ than 7, and __also equal__ to 7.

Closed circle on 7.