EPPING BOYS HIGH SCHOOL
213 Vimiera Rd Marsfield

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.
- Begin each question in a new booklet
- Ensure your name and teachers name is on every answer booklet and on this examination paper.
- This paper and all examination booklets must remain in the exam centre

Name						Teacher				
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total

Question 1 **(12 Marks)** Start a new booklet **Marks**

(a) Fully simplify $\dfrac{x}{x^2-1}+\dfrac{3}{x+1}$ **2**

(b) If $\tan\theta=\frac{7}{8}$ and , find the exact value of $\sin\theta$ **2**

(c) Solve $|16-4x|\leq 12$ **2**

(d) Find $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x-3}$ **2**

(e) Find the period and amplitude for the graph of $y=3\cos 4x$. **2**

(f) Paint at the local hardware store is sold at a profit of 30% on the cost price. If a drum of paint is sold for \$67·50, find the cost price. **2**

Question 2 **(12 Marks)** Start a new booklet **Marks**

(a) Given the points A(1,2), B(7, 8), C(-1,4)

(i) Find the equation of the line BC in general form **2**

(ii) Find the perpendicular distance from point A to the line BC. **2**

(iii) Hence, or otherwise, find the area of ΔABC. **2**

(iv) What angle does BC make with the positive direction of the *x* axis? **1**

(v) What are the coordinates of the midpoint of AB? **1**

(c) Find the value of $\log_e\left(e^2\right)+\log_e 1$ **2**

(d) Solve $\frac{1}{x}=x-1$ leaving your answer in exact form. **2**

Question 3 **(12 Marks)** Start a new booklet **Marks**

(a) Differentiate with respect to *x*.

(i) $x.e^{x}$ **2**

(ii) $\dfrac{\sin x}{x^2}$ **2**

(b) Find:

(i) $\int e^{-2x}dx$ **2**

(ii) $\int_0^{\pi} \sec^2 \dfrac{x}{4}\,dx$. **2**

(c) If α and β are the roots of the equation $2x^2 - 3x + 4$
Find:

(i) $\alpha + \beta$. **1**

(ii) $2\alpha\beta$. **1**

(iii) $\alpha^2 + \beta^2$ **2**

Question 4 **(12 Marks)** Start a new booklet **Marks**

(a) $4+4x+4x^2+4x^3+4x^4+\ldots$ is a geometric series

(i) Under what conditions does it have a limiting sum? **1**

(ii) The limiting sum is 12. What is the value of x? **2**

(b) $y=x^2-7x+19$ meets with $y=3x-6$. **2**

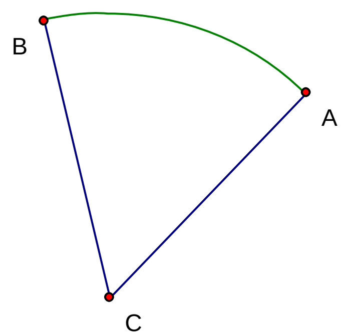
Prove $y=3x-6$ is tangent to the parabola

(c) A ship sails from port A, 80 kilometres due west, to a point B.
It then proceeds a distance of 70 kilometers on a bearing of 15°
to a point C.

(i) Draw a diagram to illustrate the information given. **1**

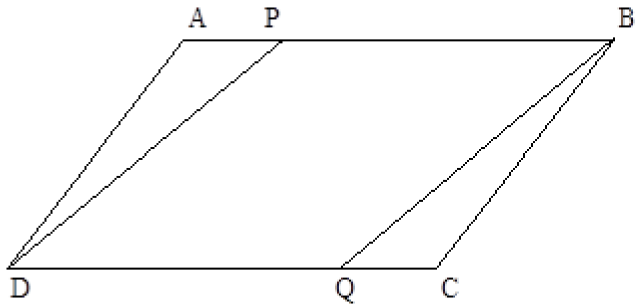
(ii) Find the distance AC **2**

(d) The sector ABC has r = 5cm
and the arc length BA = 6cm.
Calculate the area of the sector
to the nearest square centimeter. **2**



(e) Find the value of $\dfrac{2^{3n-1}}{8^{n-1}}\times\dfrac{25^m}{5^{2m+1}}$ **2**

Question 5 **(12 Marks)** Start a new booklet **Marks**



(a) ABCD is a parallelogram and BP = DQ.

(i) What can you say about BC and AD and $\angle A$ and $\angle C$ because ABCD is a parallelogram. **1**

(ii) Prove DP = BQ **3**

(b) Find the radius and centre of the circle with equation **2**

$$x^2 - 8x + y^2 = -7$$

(c) Sketch $y = \sin x$ and $y = 3\sin x + 1$ in the same set of axes $0 \le x \le 2\pi$ **3**

(d) The gradient of a function is given by $y' = \dfrac{4x}{x^2 + 1}$ **3**

The curve passes through (0, 2). What is the equation of the function?

Question 6 **(12 Marks)** Start a new booklet **Marks**

(a) A curve has a gradient function with equation $\frac{dy}{dx} = (x-1)(x+2)$.

(i) If the curve passes through the point $(1, \frac{5}{6})$, what is the equation of the curve? **2**

(ii) Find the coordinates of the stationary points and determine their nature. **2**

(iii) Find any points of inflexion. **1**

(iv) Graph the function showing all the main features. **2**

(b) Find the equation of the tangent to the curve $y = \sin 3x - 1$ at the point where $x = \frac{\pi}{12}$ **3**

Write your equation in gradient intercept form.

(c) Fully factorise $729 - 64m^6$ **2**

Question 7 **(12 Marks)** Start a new booklet **Marks**

(a) The parabola $y=4-x^2$ and the line $y=2-x$ intersect at points A and B. **3**
Find the area bounded by the parabola and the line.

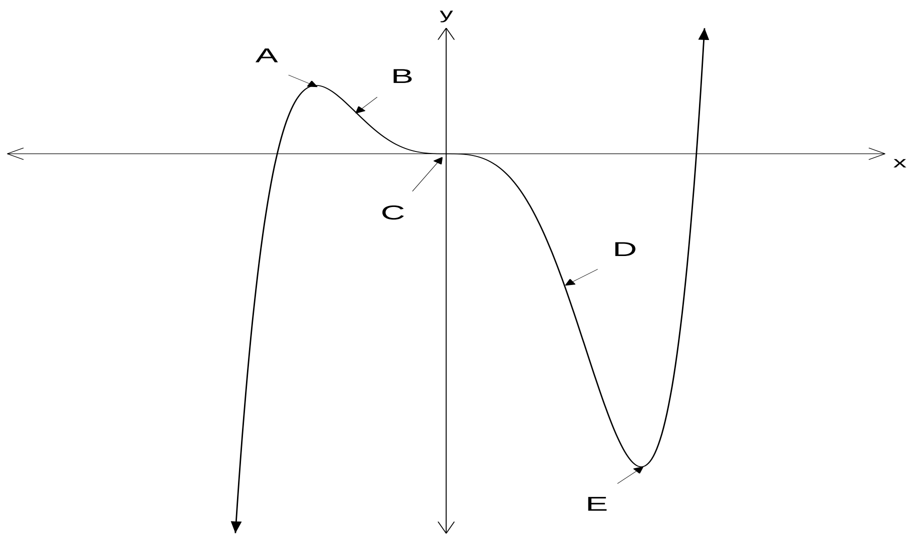
(b) Find all the solutions to $(x^2-1)^2-11(x^2-1)+24=0$ **3**

(c) Use Simpson's Rule to evaluate $\int_1^3 \log_e x^2 dx$ to 1 decimal place **3**
using 3 function values

(d) $y=2+\dfrac{1}{x}$ **3**
Find the area between the curve and the y axis between y = 4 and y = 6.

Question 8 **(12 Marks)** Start a new booklet **Marks**

(a) The graph of the curve $y = f(x)$ is drawn below, with arrows pointing to named points on the curve



(i) Name the 3 points of inflexion **1**

(ii) Describe the second derivative at the point E **1**

(iii) Where will the graph of the first derivative cross the x axis? **1**

(b) For the parabola $(x-2)^2 = 4y-12$ **3**
State the coordinates of the vertex, the focal length and the coordinates of the focus.

(c) A group sets out to establish a coffee shop in New Orleans.
Their business plan projects their anticipated income for the first 10 years.
They anticipate they can make \$72 000 in their first year and an additional \$4000 every year after that. If their business plan is correct

(i) How much will they make in their 5^{th} year? **1**

(ii) What is their total income for the first 8 years? **1**

(iii) If their business plan remains accurate, after how many years will their total income equal \$1 248 000? **2**

(d) If $y = e^{\tan x}$ find $f'\left(\dfrac{\pi}{4}\right)$ **2**

Question 9 **(12 Marks)** Start a new booklet **Marks**

(a) A particle moves in a straight line so that its displacement (in m) from a fixed point O at time t seconds is given by $t^3 - 9t - 12$

Find:

(i) The initial velocity **1**

(ii) When the particle is at rest. **1**

(iii) The distance covered in the first 4 seconds **2**

(b) The area bounded by the curve $y = \sqrt{16 - x^2}$ between the lines $x = 1$ and $x = 3$ is rotated about the x-axis. Find the volume of the solid of revolution formed. **3**

(c) On the Cartesian Plane, sketch the region satisfying the inequalities **3**

$x \geq 2$ $y \geq 4$ and $y \leq 8 - x$

(d) What is the domain and the range for $y = |\log_e(x+2)|$ **2**

Question 10 (12 Marks) Start a new booklet **Marks**

(a) An open cylindrical can with a base but no lid is made from 300cm^2 of sheet metal.

(i) Show that the volume of the can is given by $V = 150\text{r} - \frac{1}{2}\pi\text{r}^3$. **2**

(ii) Find the radius of the cylinder that gives the maximum volume. **4**

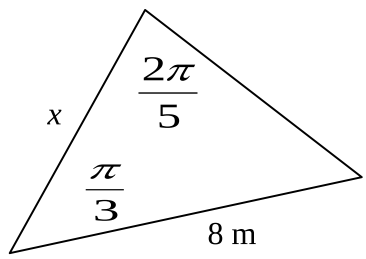
Justify that it gives a maximum and calculate this volume.

(b) Sketch $y = |x-3|$ and $y = \frac{x}{2} - 1$ on the same set of axes. **3**

Hence solve $2|x-3| > x-2$

(c) A triangular yard has its angles shown in radians. **3**

Calculate the side *x* correct to the nearest cm.



End of Examination.

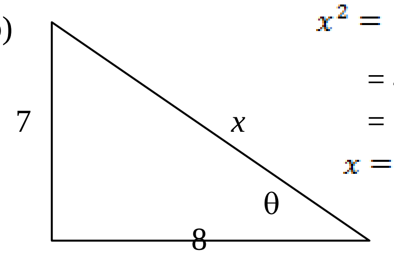
EBHS

2009

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

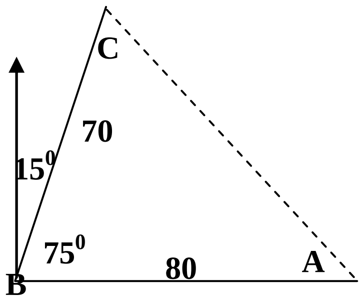
Mathematics Solutions

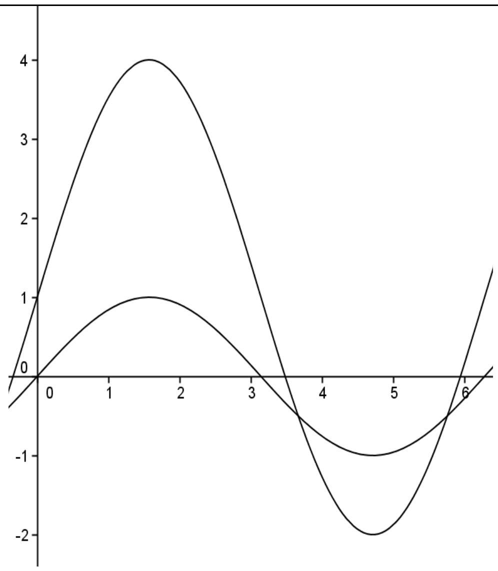
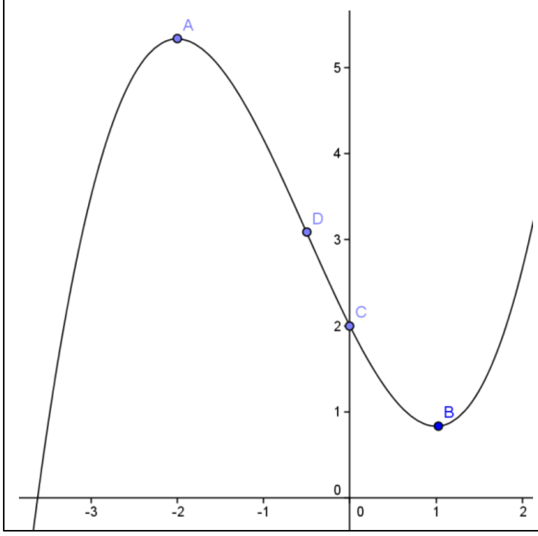
Solutions	Marks/Comments
<u>Question 1</u>	
(a) $\dfrac{x}{x^2-1}+\dfrac{3x-3}{x^2-1}=\dfrac{4x-3}{x^2-1}$	**2**
(b)  $x^2 = 7^2 + 8^2$ $= 49 + 64$ $= 113$ $x = \sqrt{113}$ Since $\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$ 3^{rd} Quadrant $\therefore \sin\theta < 0$	**1** **1**
(c) $\lvert 16-4x\rvert \le 12$ so $\lvert 4-x\rvert \le 3$ $-3 \le 4-x \le 3$ $-7 \le -x \le -1 \quad \therefore \quad 1 \le x \le 7$	**1** **1**
(d) $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x-3} = \lim\limits_{x\to 3}\dfrac{(x+1)(x-3)}{x-3}$ $\lim\limits_{x\to 3}(x+1)=4$	**1** **1**
(e) Amplitude = 3 Period = $\dfrac{2\pi}{4}=\dfrac{\pi}{2}$	**1** **1**
(f) 130% = \$67·50 1% = $\frac{67\cdot50}{130}$ = 0·51923…… Cost Price = $\frac{67\cdot50}{130} \times 100$ = \$51·92	**2 Marks - 1 for 1%** **1 for Cost price**

Solutions	Marks/Comments
Question 2 (a) (i) A(1,2), B(7, 8), C(-1,4) BC has m = ½ $y-y_1=m(x-x_1) \quad y-8=\tfrac{1}{2}(x-7) \quad 2y-16=x-7$ $0=x-2y+9$	1 for method 1 for correct answer
(ii) $d=\dfrac{\lvert Ax_1+By_1+C\rvert}{\sqrt{A^2+B^2}} = \dfrac{\lvert 1\times1-2\times2+9\rvert}{\sqrt{5}}=\dfrac{6\sqrt{5}}{5}$	1 for sub, 1 for get rationalising denom not req[d] Award full marks for correct use of incorrect equation for part i
(iii) $BC=\sqrt{8^2+4^2}=\sqrt{80}=4\sqrt{5} \quad \therefore A=\dfrac{1}{2}\times\dfrac{6\sqrt{5}}{5}\times4\sqrt{5}=12$	2 allow approximation
(iv) $m_{BC}=\dfrac{1}{2} \quad \tan^{-1}(.5)=26.565\cong27^\circ$	1
(v) midpoint = (4, 5)	1
(b) 2 + 0 = 2	2 allow 1 for part correct
(c) $1=x^2-x \quad 0=x^2-x-1$ $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{1\pm\sqrt{5}}{2}$	1 for the eqn and Q form 1 for the get /12

Solutions	Marks/Comments
Question 3 i. $\dfrac{d}{dx}\left(x.e^x\right)=vu'+uv'=e^x+xe^x=e^x(1+x)$	**2 factorising not required**
ii. $\dfrac{d}{dx}\dfrac{\sin x}{x^2} = \dfrac{vu'-uv'}{v^2}=\dfrac{x^2\cos x-2x\sin x}{x^4}$	**2**
(b) i. $\int e^{-2x}dx=-\dfrac{1}{2}e^{-2x}+c$	**2 must have c**
ii. $\int_0^\pi \sec^2\dfrac{x}{4}dx=4\left[\tan\dfrac{x}{4}\right]_0^\pi$ $=4\left[\tan\dfrac{\pi}{4}-\tan 0\right]=4$	**2**

(c) i. 1.5	**1**
ii. $2\alpha\beta = \frac{2c}{a} = 4$	
iii. $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 2.25-4 = -1.75$	**1**
	2

Solutions	Marks/Comments
<u>Question 4</u>	
a) i) $\lvert x\rvert < 1$ or $-1 < x < 1$	**1**
ii) $S_\infty = 12 = \frac{a}{1-r} = \frac{4}{1-x}$	**1**
$1-x = \frac{4}{12}$ $x = \frac{2}{3}$	**1**
(b) $x^2-7x+19 = 3x-6$ $x^2-10x+25=0$ $\therefore$ 1 root and tangent	**1**
$b^2-4ac = 100-4\times 25 = 0$	**1**
(c) 	
$a^2 = b^2+c^2-2bc\cos A$	
$= 70^2+80^2-2\times 70\times 80\cos 75$	**1** For diagram
$= 8401.2266...$	
$a = 91.66m$	**2**
(d) $\theta = 6/5 = 1.2^c$	
$A = \frac{\theta}{2}r^2$	**1**
$= .6\times 25 = 15cm^2$	**1**
e) $\frac{2^{3n-1}}{8^{n-1}}\times\frac{25^m}{5^{2m+1}} = \frac{2^{3n-1}}{2^{3(n-1)}}\times\frac{5^{2m}}{5^{2m+1}} = \frac{2^2}{5} = 0.8$	**2**
<u>Question 5</u>	**1 for correct diagram**
i. BC = AD and ∠A = ∠C property of parallelogram	**1**
ii. QC = AP since CD = AB and DQ = PB given	**1**
$\Delta DAP \equiv \Delta BCQ$ SAS rule	**1**
∴ DP = BQ (corresponding sides in congruent triangles)	**1**
(b) $x^2-8x+y^2 = -7$ $x^2-8x+16+y^2 = -7+16$	**1**
$(x-4)^2+y^2 = 9$ centre (4, 0) radius3	**1**

(c) 	**1** **1** **1**
(d) $y' = \dfrac{4x}{x^2+1} \quad y = \displaystyle\int \dfrac{4x\,dx}{x^2+1} = 2\ln(x^2+1) + c$ But when x = 0, y = 2 whence c = 2 and $y = 2\ln(x^2+1) + 2 = \ln(x^2+1)^2 + 2$	**1** **2** **/12**
Question 6 (a) $\dfrac{dy}{dx} = (x-1)(x+2) = x^2 + x - 2$ $y = \displaystyle\int x^2 + x - 2dx = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x + c$	**1 for correct integration**
But when x = 1, y = $\dfrac{5}{6}$ $\quad \dfrac{5}{6} = \dfrac{1}{3} + \dfrac{1}{2} - 2 + c$ whence c = 2 $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 2x + 2$	**1 for finding *c* CFPA**
ii) If $y' = 0,\ x = 1\ or\ -2$ $\quad \left(1, \dfrac{5}{6}\right)$ is a known point, $f(-2) = \dfrac{-8}{3} + 2 + 4 + 2 = 5\dfrac{1}{3}$ so $\left(-2,\ 5\dfrac{1}{3}\right)$ is the second Stat. Point $f''(x) = 2x + 1 \quad f''(-2) = -3 \quad \therefore$ MAX $\quad f''(1) = 3 \quad \therefore$ MIN	**1 for both *x* values** **1 y coordinates and justification**
 iii) $2x + 1 = 0 \quad x = -\dfrac{1}{2}$ $f\left(-\dfrac{1}{2}\right) = -\dfrac{1}{24} + \dfrac{1}{8} + 1 + 2 = 3\dfrac{1}{12}$ $\left(-\dfrac{1}{2},\ 3\dfrac{1}{12}\right)$ is P of I iv) noting y intercept = 2	**1 finding *x* coordinate sufficient** **2 detail required correct from their working**

b) $y=\sin 3x-1 \quad y'=3\cos 3x \quad f'\left(\frac{\pi}{12}\right)=3\cos\frac{\pi}{4}=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}\approx 2.12$

required m

1

$f\left(\frac{\pi}{12}\right)=\sin\frac{\pi}{4}-1=\frac{\sqrt{2}}{2}-1\approx -.29$ y coordinate of point

$$y+1-\frac{\sqrt{2}}{2}=\frac{3\sqrt{2}}{2}\left(x-\frac{\pi}{12}\right)$$

1

$y-y_1=m(x-x_1) \quad 2y+2-\sqrt{2}=3\sqrt{2}x-\frac{\sqrt{2}\pi}{4}$ OK to use 0.26 as y_1

$$y=\frac{3\sqrt{2}}{2}x-\frac{\sqrt{2}\pi}{8}-1+\frac{\sqrt{2}}{2}$$

1

In approximate form $y=2.12x-.85$ would be adequate

Approximate form is OK since exact form was not stated

c) $729-64m^6=(9-4m^2)(81+36m^2+16m^4)$

$=(3+2m)(3-2m)(81+36m^2+16m^4)$

1

1 **OK not to have factorised** $81+36m^2+16m^4$

/12

Alternatively $729-64m^6=(27-8m^3)(27+8m^3)$

$=(3-2m)(9+6m+4m^2)(3+2m)(9-6m+4m^2)$

Is a more elegant solution

Question7

a) $4-x^2=2-x \quad 0=x^2-x-2=(x-2)(x+1)$ *so* $x=2$ *or* -1

1

$$A=\int_{-1}^{2}\left|2+x-x^2dx\right|=\left[2x+\frac{x^2}{2}-\frac{x^3}{3}\right]_{-1}^{2}=4+2-\frac{8}{3}-\left(-2+\frac{1}{2}+\frac{1}{3}\right)=4\frac{1}{2}$$

2

b) $(x^2-1)^2-11(x^2-1)+24=0 \quad u^2-11u+24=(u-3)(u-8)=0$

$x^2-1=3 \ x^2=4 \ x=\pm 2 \quad or \quad x^2-1=8 \ x^2=9 \ x=\pm 3$

1

2 allow 1 for 2, 3 = x

c) $\int_1^3 \log_e x^2 dx \approx \frac{h}{3}(d_F+4d_M+d_L)=\frac{1}{3}((\ln(1)+4\ln(4)+\ln(9))=2.58....$

3

d) $y=2+\frac{1}{x}$ *becomes* $x=\frac{1}{y-2}$

$\int_4^6 \frac{dy}{y-2}dx=[\log_e(y-2)]_4^6=\log_e 4-\log_e 2=\log_e 2$

3

Question 8

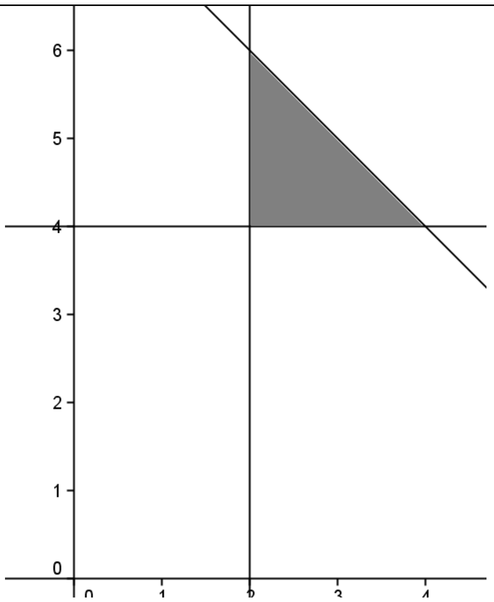
a) i) B, C, D **1**

ii) $f''(x)>0$ **1**

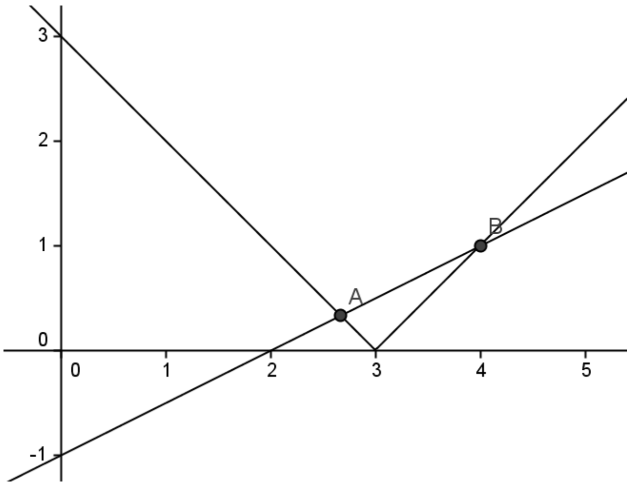
iii) at the points A, C and E **1**

b) $(x-2)^2 = 4y-12$ *ie* $(x-2)^2 = 4(y-3)$ vertex is (2, 3) focal length = 1 focus = (2, 4)	**1** **2**
c) i) $a = 72000,\ d = 4000\quad T_5 = 72000 + 4\times 4000 = \$88000$	**1**
ii) $S_8 = \frac{8}{2}\left(2\times 72000 + 7\times 4000\right)$ = \$688 000	**1**
$S_n = 1248 = \frac{n}{2}(72\times 2 + (n-1)4)$	**1**
iii) $1248\times 2 = 144n + 4n^2 - 4n\quad n^2 + 35n - 624 = 0$ $(n-13)(n+48) = 0$ The amount is earned over the course of 13 years	**1 only award 1 if both answers for n are given**
d) $y = e^{\tan x}\quad y' = \sec^2 x e^{\tan x}$	**1**
$f'\left(\frac{\pi}{4}\right) = 2e^1 = 2e$	**1** **/12**

Question 9	
a) i) $x = t^3 - 9t - 12\quad \frac{dx}{dt} = 3t^2 - 9\quad v(0) = 9$	**1**
ii) $3t^2 - 9 = 0$ gives $t = \pm\sqrt{3}$	**1**
iii) $d = \int_0^{\sqrt{3}} 3t^2 - 9dt + \int_{\sqrt{3}}^{4} 3t^2 - 9dt$	**1**
$d = \left\|\left[t^3 - 9t\right]_0^{\sqrt{3}}\right\| + \left\|\left[t^3 - 9t\right]_{\sqrt{3}}^{4}\right\| = \left\|3\sqrt{3} - 9\sqrt{3}\right\| + \left\|64 - 36 - (3\sqrt{3} - 9\sqrt{3})\right\|$ $= 28 + 12\sqrt{3}m$	**1**
Alternatively $x(0) = -12\quad x(\sqrt{3}) = -6\sqrt{3} - 12\quad x(4) = 64 - 36 - 12 = 16$ and d = 28+12 $\sqrt{3}$	
b) $V = \pi\int_1^3 y^2 dx = \pi\int_1^3 (16 - x^2)dx = \pi\left[16x - \frac{x^3}{3}\right]_1^3 = \pi\left(48 - 9 - \left(16 - \frac{1}{3}\right)\right)$ $= \frac{70\pi}{3}u^3$	**3** **Exact form not required**

c) 	**2**
d) D $x > -2$ R $y \geq 0$	**1 + 1**

Solutions	Marks/Comments
<u>Question 10</u>	
(a) i. SA = $\pi r^2 + 2\pi r h = 300$ $2\pi r h = 300 - \pi r^2$ $h = \frac{300 - \pi r^2}{2\pi r}$ $V = \pi r^2 h$ $= \cancel{\pi r^2}\left(\frac{300 - \pi r^2}{\cancel{2\pi r}}\right)$ $= 150r - \frac{\pi r^3}{2}$	**2 marks – 1 for "h"** **1 for "V"**
ii. $V = 150r - \frac{1}{2}\pi r^3$ $\dot{V} = 150 - \frac{3}{2}\pi r^2$ $\ddot{V} = -3\pi r$ which is less than 0 for positive r	**4 marks – 1 for differentials** **1 for value of 'r'** **1 for test** **1 for max volume**
Stat Pts when $\dot{V} = 0$ i.e. $150 - \frac{3}{2}\pi r^2 = 0$ $150 = \frac{3}{2}\pi r^2$ $100 = \pi r^2$ $r^2 = \frac{100}{\pi}$ $r^2 = \pm\sqrt{\frac{100}{\pi}}$ Now max Volume when $r > 0$ i.e. $r = \sqrt{\frac{100}{\pi}} = 5{\cdot}641895835$ or $\frac{10}{\sqrt{\pi}}$	

$V = 150\sqrt{\frac{100}{\pi}} - \frac{\pi}{2}\left(\sqrt{\frac{100}{\pi}}\right)^3 = 564{\cdot}34$ $= 564\text{cm}^3$ (nearest cm^3)		
 (b)	1	
A is the intersection of $3 - x = \frac{x}{2} - 1 \quad x = 2\frac{2}{3}$	1	
B is the intersection of $x - 3 = \frac{x}{2} - 1 \quad x = 4$	1	
$2\lvert x-3\rvert > x-2$ has solution $x < 2\frac{2}{3}$ *OR* $x > 4$	1	
c) Remaining angle = $\frac{4\pi}{15}$	2	/12
$\frac{x}{\sin\frac{4\pi}{15}} = \frac{8}{\sin\frac{2\pi}{5}} \qquad x = 8\sin\frac{4\pi}{15} \div \sin\frac{2\pi}{5} = 6.25m$		