Penrith Selective
High School

2011
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- All questions are of equal value
- Staple this test to your answers
- Attempt Questions 1 – 10

Total marks – 120

Question	Mark
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
8	/12
9	/12
10	/12
Total	/120
Percentage	

Student's Name: ______________________________

Teacher: ______________________________

Question 1 **(Start a new page)**

a) The line $3x + ky = 11$ passes through the point (2, -1). Find the value of k. **(2)**

b) Simplify $\frac{m^2 - 3m}{m^2 - 9}$. **(2)**

c) If $n = 1.6 \times 10^4$, write $\frac{1}{n}$ in scientific notation. **(2)**

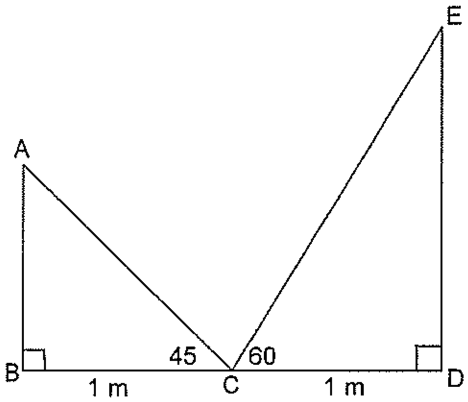
d) Solve $x^{\frac{3}{2}} = 5$, correct to 2 decimal places. **(2)**

e) Find the integers a and b such that $\frac{1}{\sqrt{15}+4} = a\sqrt{15} + b$. **(2)**

f) If α and β are the roots of $3x^2 + 2x - 9 = 0$, what is the value of $\frac{\alpha\beta}{\alpha+\beta}$? **(2)**

Question 2 **(Start a new page)**

a)



i. What are the exact lengths of AC and EC. **(2)**

ii. Find the length of AE to the nearest centimetre. **(2)**

b) Prove that $\frac{\text{cosec}^2\,\theta - \cot^2\theta}{\cot\theta\,\sin\theta} = \sec\theta$. **(2)**

c) A parabola P has equation $x^2 = 4(3 - y)$. **(3)**

State

i. the coordinates of its vertex
ii. the equation of its directrix
iii. the coordinates of its focus
iv. its focal length.

Draw a neat sketch of P and clearly indicate on it the above features.

d) Consider the equation $x^2 + (k-4)x + 9 = 0$. **(3)**
For what values of k does the equation have distinct real roots?

Question 3 **(Start a new page)**

a) Differentiate:

i. $\frac{1}{3x^3}$ **(1)**

ii. $(x^3 + 4)^5$ **(2)**

iii. $\frac{x^2}{1+x^2}$ **(2)**

b) Evaluate

i. $\int_0^6 (x^2 + x + 1)\, dx$ **(2)**

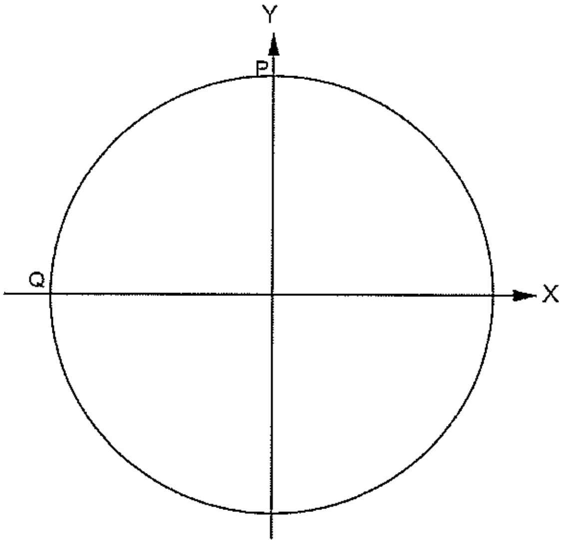
ii. $\int_1^9 x\sqrt{x}\, dx$ **(2)**

c) Find the exact volume generated when the region bounded by the curve $y = 6x - x^2$ and the x axis is rotated about the x axis. **(3)**

Question 4 **(Start a new page)**

The diagram below shows the circle $x^2 + y^2 = 25$.

P is the point where it meets the y axis and Q is a point where it meets the x axis, as shown.



Copy or trace the diagram on your answer page.

a) State the co-ordinates of P and Q. **(1)**

b) Prove that R(3, 4) also lies on the circle and mark it on the diagram. **(1)**

c) Find the gradient of PR. **(1)**

d) M is the midpoint of PR and O is the origin.
Prove that OM and PR are perpendicular. **(2)**

e) Show that the equation of PR is
$x + 3y - 15 = 0$. **(1)**

f) Find the perpendicular distance between Q and the line PR. **(2)**

g) Find the area of ΔPQR. **(2)**

h) On the diagram shade the region in which $x^2 + y^2 \leq 25$ and $x + 3y - 15 \geq 0$ are both true. **(2)**

Question 5 **(Start a new page)**

a) If $\log_5 x + 2\log_5 y = 2$, express x in terms of y. **(2)**

b) Find the primitive function of: **(4)**

i. $\frac{8x}{1+2x^2}$

ii. $4e^{6x}$

c) Solve the following equation for x. **(2)**

$$e^{2x} + e^{x} - 12 = 0.$$

d) i. On the same diagram sketch the curves $y = e^{x}$ and $y = e^{-x}$. **(2)**

Clearly label any the important features

ii. Calculate the exact area bounded by the curves $y = e^{x}, y = e^{-x}$ and the line $x = 4$. **(2)**

Question 6 **(Start a new page)**

a) Find $\int(\sin 3x)\,dx$ **(1)**

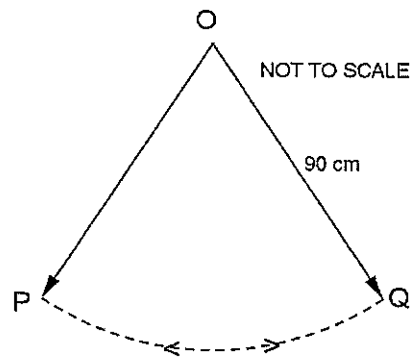
b) i. Differentiate $\sin^4 x$. **(2)**

ii. Hence evaluate $\int_0^{\frac{\pi}{2}}(\sin^3 x\cos x)dx$ **(2)**

c) Find the equation of the normal to $y = x\sin x$ at the point where $x = \frac{\pi}{2}$. **(3)**

d) Sketch the curve $y = 3\cos 2x \quad 0 \le x \le \pi$ **(2)**

e) A pendulum is 90 cm long and swings through a sector whose extreme positions are indicated by the points P and Q.



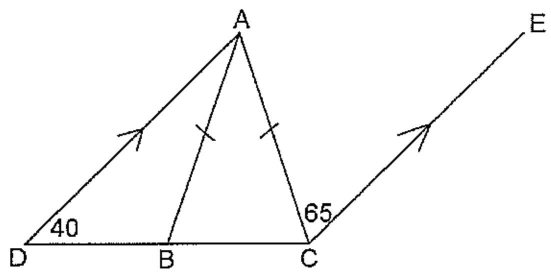
If the area of the sector swept out by the pendulum is 2430 cm^2.

Find the length of the arc PQ. **(2)**

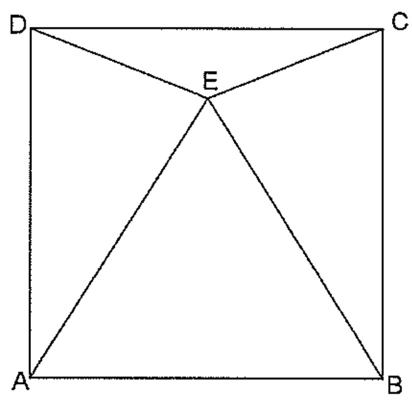
Question 7 **(Start a new page)**

a) Use Simpson's Rule with three function values to find a two decimal approximation to $\int_1^3 \sqrt{x + \frac{3}{x}}\, dx$. **(2)**

b) In the diagram below AD || CE, angle ADB = 40°, angle ACE = 65° and AB = AC. Find angle BAC giving reasons. **(3)**



c) ABCD is a square. ΔABE is equilateral.



a. Prove angle EBC = 30°. **(2)**

b. Prove triangles EBC and EAD are congruent. **(2)**

c. If the lengths of the sides of the square are k, prove that the area of CDE is $\frac{k^2(2-\sqrt{3})}{4}$ cm^2. **(3)**

Question 8 **(Start a new page)**

a) If $y = \log_e(x^2 + 1)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **(2)**

b) Show that $y = \log_e(x^2 + 1)$ is an even function. **(2)**

c) Show that there is one stationary point and determine its nature. **(2)**

d) Find any points of inflexion. **(2)**

e) Sketch the curve, showing any important features. **(2)**

f) Find the positive values of x for which the quantity y is increasing at a decreasing rate. **(2)**

Question 9 **(Start a new page)**

a) i. Given that $a^2 + b^2 = 11ab$, use this result to show that $\left(\frac{a-b}{3}\right)^2 = ab$. **(1)**

ii. Hence write $\log\left(\frac{a-b}{3}\right) - \frac{1}{2}(\log a + \log b)$ in simplest form. **(2)**

b) A biased coin is tossed n times. If the probability of getting a head is $\frac{3}{5}$,

i. find the probability of getting n tails. **(1)**

ii. find the probability of getting at least one head in n tosses. **(1)**

iii. what is the least number of times would you have to toss the coin so that the probability of getting at least one head is greater than 99%? **(2)**

c) One bag contains three black marbles and one white marble. Another bag contains two black marbles and three white marbles. Tamzid takes one marble at random from each bag and places them in a third bag.

i. What is the probability that the third bag contains

1. Two black marbles? **(1)**
2. One white and one black marble? **(2)**

ii. If one ball is then taken at random from the third bag, what is the probability that it is black? **(2)**

Question 10 **(Start a new page)**

a) The sum of the first n terms of a certain arithmetic series is given by:

$$S_n = \frac{n(3n+7)}{2}.$$

i. Calculate S_1 and S_2. **(2)**

ii. Find the first three terms of the sequence. **(2)**

iii. Find and expression for the nth term of the sequence. **(1)**

b) An infinite geometric series has a first term of 4, a common ratio of r, and a limiting sum of $25r$. Find the value(s) of r. **(2)**

c) At the beginning of each year, Priyanka will deposit \$10 000 in a fund paying 10% per year compound interest. Interest is calculated annually.

i. What will be the value of Priyanka's first deposit when she has invested it for n years? **(1)**

ii. Show that when her first deposit has been invested for n years the total value of all her deposits will be:

$$\$\frac{10000(1.1)(1.1^n-1)}{0.1}$$ **(2)**

iii. Priyanka will withdraw all her funds when their total value exceeds \$550 000. How many deposits will she make? **(2)**

End Of Paper.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a \neq 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

2011 MATHEMATICS TRIAL SOLUTIONS.

Q1 a) $3x + ky = 11; (2, -1)$

$3(2) + k(-1) = 11$

$6 - k = 11$

$-k = 5$

$k = -5$

b) $\frac{m^2-3m}{m^2-9} = \frac{m(m-3)}{(m+3)(m-3)}$

$= \frac{m}{m+3}$

c) $\frac{1}{1 \cdot 6 \times 10^4} = 6 \cdot 25 \times 10^{-5}$

d) $x^{3/2} = 5$

$x^3 = 5^2$

$x = \sqrt[3]{25}$

$= 2 \cdot 92$

e) $\frac{1}{\sqrt{15}+4} = \frac{1}{\sqrt{15}+4} \times \frac{\sqrt{15}-4}{\sqrt{15}-4}$

$= \frac{\sqrt{15}-4}{15-16}$

$= \frac{\sqrt{15}-4}{-1}$

$= -\sqrt{15}+4$

$\therefore a = -1 \quad b = 4$

f) $\alpha + \beta = \frac{-b}{a} = \frac{-2}{3}$

$\alpha\beta = \frac{c}{a} = \frac{-9}{3} = -3$

$\frac{\alpha\beta}{\alpha+\beta} = \frac{-3}{-2/3}$

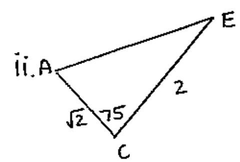
$= \frac{9}{2}$

$= 4\frac{1}{2}$

2ai $AC = \sqrt{2}$

$EC = 2$

ii.



$AE^2 = 2 + 4 - 2\sqrt{2} \cdot 2\cos 75$

$AE = 2{\cdot}13\text{cm}$

b. $\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\cot\theta \sin\theta} = \sec\theta$

$LHS = \frac{1+\cot^2\theta - \cot^2\theta}{\frac{\cos\theta}{\sin\theta}\sin\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta = RHS$

c. $x^2 = 4(3-y)$

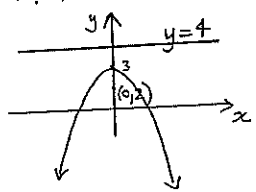
$x^2 = -4(y-3)$

i. vertex $(0,3)$

ii. directrix $y=4$

iii. focus $(0,2)$

iv. 1



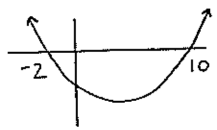
d. distinct real roots

$\Delta > 0$

$(k-4)^2 - 4.1.9 > 0$

$k^2 - 8k - 20 > 0$

$(k-10)(k+2) > 0$



$k < -2 \quad k > 10$

Q3

a) (i) $\frac{d}{dx}\left(\frac{1}{3x^3}\right) = \frac{d}{dx}\left(\frac{1}{3}x^{-3}\right)$

$= \frac{1}{3} \times -3x^{-4}$

$= -x^{-4}$

$= \frac{-1}{x^4}$

(ii) $\frac{d}{dx}(x^3+4)^5 = 5(x^3+4)^4 . 3x^2$

$= 15x^2(x^3+4)^4$

(iii) $\frac{d}{dx}\left(\frac{x^2}{1+x^2}\right) = \frac{(1+x^2)2x - x^2(2x)}{(1+x^2)^2}$

$= \frac{2x+2x^3-2x^3}{(1+x^2)^2}$

$= \frac{2x}{(1+x^2)^2}$

b (i) $\int_0^6 (x^2+x+1)\,dx = \left[\frac{x^3}{3}+\frac{x^2}{2}+x\right]_0^6$

$= \left(\frac{6^3}{3}+\frac{6^2}{2}+6\right) - 0$

$= 72+18+6$

$= 96$

(ii) $\int_1^9 x\sqrt{x}\,dx = \int_1^9 x^1 . x^{1/2}\,dx$

$= \int_1^9 x^{3/2}\,dx$

$= \left[\frac{x^{5/2}}{5/2}\right]_1^9$

$= \left[\frac{2}{5}x^{5/2}\right]_1^9$

$= \frac{2}{5}\times 9^{5/2} - \frac{2}{5}\times 1^{5/2}$

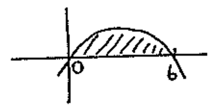
$= \frac{486}{5} - \frac{2}{5}$

$= \frac{484}{5}$

c) $6x - x^2 = 0$

$x(6-x) = 0$

$x = 0, 6$



$V = \pi\int y^2\,dx$

$= \pi\int_0^6 (6x-x^2)^2\,dx$

$= \pi\int_0^6 (36x^2-12x^3+x^4)\,dx$

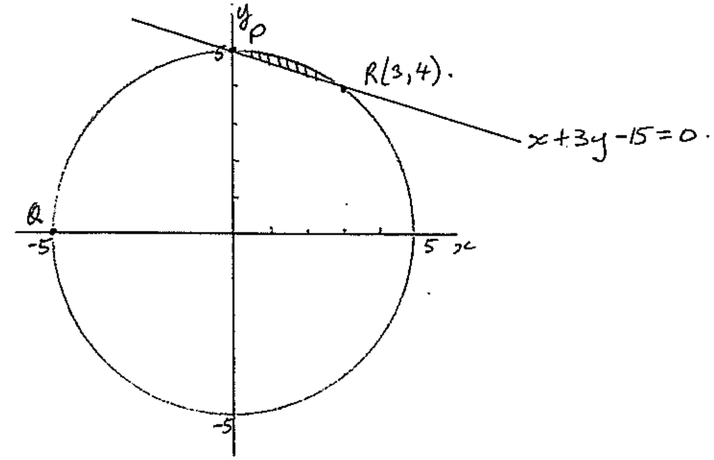
$= \pi\left[12x^3 - 3x^4 + \frac{x^5}{5}\right]_0^6$

$= \pi\left[\left(12\times6^3-3\times6^4+\frac{6^5}{5}\right)-0\right]$

$= \pi\left(\frac{1296}{5}\right)$

$= \frac{1296\pi}{5}$ or $259\frac{1}{5}\pi$

Q4



a) $P(0,5)$ + $Q(-5,0)$

b) $(3,4)$ $x^2+y^2 = 25$

$3^2+4^2 = 25$

$9+16 = 25$

c) $m_{PR} = \frac{4-5}{3-0} = -\frac{1}{3}$

d) $M = \left(\frac{0+3}{2}, \frac{5+4}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

$m_{OM} = \frac{\frac{9}{2}-0}{\frac{3}{2}-0} = 3$

$m_{PR} \times m_{OM} = -\frac{1}{3}\times 3 = -1$

$\therefore PR \perp OM$

e) $y - 5 = -\frac{1}{3}(x-0)$

$3y - 15 = -x$

$x+3y-15 = 0$

f) $d = \left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$= \left|\frac{-5+3(0)-15}{\sqrt{1^2+3^2}}\right|$

$= \left|\frac{-20}{\sqrt{10}}\right|$

$= \frac{20}{\sqrt{10}} = 2\sqrt{10}$

g) $d_{PR} = \sqrt{(3-0)^2+(4-5)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$A = \frac{1}{2}\ b\ h$

$= \frac{1}{2}\ \sqrt{10}\times 2\sqrt{10}$

$= 10\,u^2$

h) $x+3y-15=0$ correctly positioned + area shaded on diagram.

5. a) $\log_5 x + \log_5 y^2 = 2$

$\log_5 (xy^2) = 2$

$5^2 = xy^2$

$x = \frac{25}{y^2}$

b) i) $\int \frac{8x}{1+2x^2}\,dx$

$= 2\int \frac{4x}{1+2x^2}\,dx$

$= 2\ln(1+2x^2) + C$

(ii) $\int 4e^{6x}\,dx =$

$4 \times \frac{e^{6x}}{6} = \frac{2}{3}e^{6x} + C$

c) $e^{2x} + e^{x} - 12 = 0$

$(e^x)^2 + e^x - 12 = 0$

let $U = e^x$

$U^2 + U - 12 = 0$

$(U+4)(U-3) = 0$

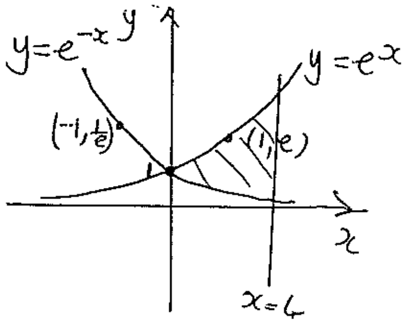
$U = 3$ or -4

$e^x = -4$ No solns

or $e^x = 3$

$\therefore x = \ln 3$

d)



(ii) $A = \int_0^4 e^x - e^{-x}\,dx$

$= \left[e^x + e^{-x}\right]_0^4$

$= \left(e^4 + \frac{1}{e^4}\right) - (1+1)$

$= \left(e^4 + \frac{1}{e^4} - 2\right)$ unit^2

Q6

a) $\int \sin 3x\,dx = -\frac{1}{3}\cos 3x + C$

b)(i) $\frac{d}{dx}(\sin^4 x) = \frac{d}{dx}(\sin x)^4$

$= 4(\sin x)^3 \cdot \cos x$

$= 4\sin^3 x \cos x$

(ii) $\int_0^{\pi/2} \sin^3 x \cos x\,dx = \frac{1}{4}\int_0^{\pi/2} 4\sin^3 x \cos x\,dx$

$= \frac{1}{4}\left[\sin^4 x\right]_0^{\pi/2}$

$= \frac{1}{4}\left[\left(\sin\frac{\pi}{2}\right)^4 - (\sin 0)^4\right]$

$= \frac{1}{4}(1-0)$

$= \frac{1}{4}$

c) $y = x\sin x$

$\frac{dy}{dx} = x\cos x + 1 \times \sin x$

$= x\cos x + \sin x$

At $x = \frac{\pi}{2}$.

$y = \frac{\pi}{2}\sin\frac{\pi}{2}$

$= \frac{\pi}{2}$

$\frac{dy}{dx} = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}$

$= 0 + 1$

$= 1$

$M_T = 1$

$M_N = -1$

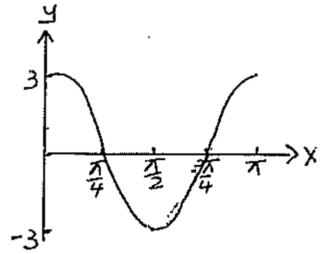
$\therefore y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$

$y - \frac{\pi}{2} = -x + \frac{\pi}{2}$

$x + y = \pi$

$x + y - \pi = 0$

d) $y = 3\cos 2x$



e) $A = \frac{1}{2}r^2\theta$

$2430 = \frac{1}{2} \times 90^2 \times \theta$

$0.6 = \theta$

$l = r\theta$

$= 90 \times 0.6$

$= 54$ cm

7a

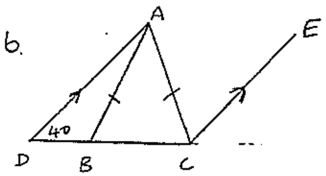
x	1	2	3
$f(x)$	2	$\sqrt{\frac{7}{2}}$	2

$A \doteq \frac{b-a}{6}\left\{f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right\}$

$= \frac{2}{6}\left\{2+4\sqrt{\frac{7}{2}}+2\right\}$

$= 3.83$

b.



$\angle ECD = 140$ (cointerior ∠'s AD∥EC)

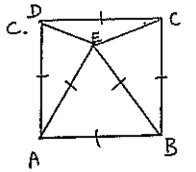
$\angle ACB = 140-65$ (adjacent ∠'s)

$= 75°$

$\angle CAB = 180 - 2\times 75$ (∠sum isos Δ)

$= 30°$

c.



a. Prove that $\angle EBC = 30°$

$\angle CBA = 90$ (∠ of a square)

$\angle EBA = 60$ (∠ of equilateral Δ)

$\therefore \angle EBC = 90-60 = 30°$

b. Prove $\Delta EBC \equiv \Delta EAD$

$\angle EBC = 30°$ (part a)

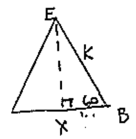
Similarly $\angle DAE = 30°$ (A)

$AD = CB$ (sides of a square) (S)

$EA = EB$ (sides of a equilateral Δ) (S)

$\therefore \Delta EDC \equiv \Delta EAD$

d. A line is drawn from E meeting AB at X perpendicular to AB.
A line is drawn from E meeting DC at Y perpendicular to DC.

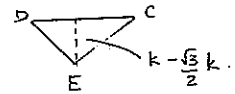


$\sin 60 = \frac{EX}{k}$

$EX = k\sin 60$

$= \frac{\sqrt{3}}{2}k$

$EY = k - \frac{\sqrt{3}}{2}k$



Area $\Delta DCE = \frac{1}{2}k\left(k-\frac{\sqrt{3}k}{2}\right)$

$= \frac{1}{2}k^2\left(1-\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}k^2\left(\frac{2-\sqrt{3}}{2}\right)$

$= \frac{k^2(2-\sqrt{3})}{4}\ cm^2$

Q8

a) $y = \log_e(x^2+1)$

$\frac{dy}{dx} = \frac{2x}{x^2+1}$

$\frac{d^2y}{dx^2} = \frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2}$

$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$= \frac{-2x^2+2}{(x^2+1)^2}$

b) Even function $f(a) = f(-a)$

$f(a) = \ln(a^2+1)$

$f(-a) = \ln((-a)^2+1)$

$= \ln(a^2+1)$

$= f(a)$

∴ even function

c) Stationary Point at $\frac{dy}{dx}=0$

$\frac{2x}{x^2+1} = 0$

As $x^2+1 \neq 0$

$\therefore 2x = 0$

$x = 0$

when $x=0$

$\frac{d^2y}{dx^2} = \frac{-2(0)+2}{(0+1)^2} = 2$

∴ minimum as $\frac{d^2y}{dx^2} > 0$ at $(0,0)$

d) Pts of inflexion $\frac{d^2y}{dx^2}=0$

$\frac{2-2x^2}{(x^2+1)^2} = 0$

$\therefore 2-2x^2 = 0$

$2x^2 = 2$

$x^2 = 1$

$x = \pm 1$

At $x=1$

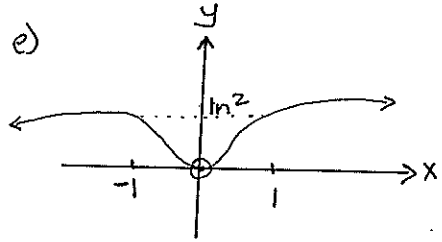
x	0	1	2
y''	2	0	$-6/25$

∴ change in concavity

∴ pt of inflexion at $(1, \ln 2)$

Also a pt of inflexion at $(-1, \ln 2)$ as it is an even function

e)



f) $\frac{dy}{dx} > 0 \quad \frac{d^2y}{dx^2} < 0$

$\therefore x > 1$

(Q9)

a) i) $a^2+b^2=11ab.$

$$\left(\frac{a-b}{3}\right)^2 = \frac{a^2-2ab+b^2}{9}$$
$$= \frac{11ab-2ab}{9}$$
$$= \frac{9ab}{9}$$
$$= ab.$$

ii) $\therefore \frac{a-b}{3} = (ab)^{\frac{1}{2}}$

$$\log\left(\frac{a-b}{3}\right) - \frac{1}{2}(\log a + \log b)$$
$$= \log(ab)^{\frac{1}{2}} - \frac{1}{2}(\log ab)$$
$$= \frac{1}{2}\log(ab) - \frac{1}{2}\log(ab)$$
$$= 0$$

b) $P(h) = \frac{3}{5}$, $Pr(t) = \frac{2}{5}$.

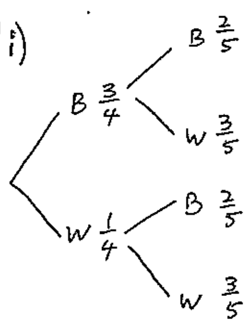
i) $P(n \times t) = \left(\frac{2}{5}\right)^n$

ii) $Pr(\text{at least } 1h) = 1 - \left(\frac{2}{5}\right)^n$

iii) $1-\left(\frac{2}{5}\right)^n \geq 0.99$

$$-\left(\frac{2}{5}\right)^n \geq -0.01$$
$$\left(\frac{2}{5}\right)^n \leq 0.01$$
$$n\log\left(\frac{2}{5}\right) \leq \log(0.01)$$
$$n \geq \frac{\log(0.01)}{\log(0.4)}$$
$$n \geq 5.03$$
$$\therefore n = 6$$

c) i)



1) $P(BB) = \frac{3}{4} \times \frac{2}{5} = \frac{3}{10}.$

2) $P(B+W) = Pr(BW)$ or $Pr(WB)$

$$= \frac{3}{4}\times\frac{3}{5} + \frac{1}{4}\times\frac{2}{5}$$
$$= \frac{9}{20} + \frac{2}{20}$$
$$= \frac{11}{20}$$

ii) from $(B\bar{B}) \rightarrow Pr(\bar{B}) = \frac{3}{10} \times 1 = \frac{3}{10}.$

OR from $(B+W) \rightarrow Pr(\bar{B}) = \frac{11}{20} \times \frac{1}{2} = \frac{11}{40}$

$\therefore Pr(\bar{B}) = \frac{3}{10} + \frac{11}{40} = \frac{23}{40}$

Note:
from $(WW) \rightarrow Pr(\bar{B}) = 0$

10.a.

i. $S_1 = \frac{1(10)}{2} = 5$

$S_2 = \frac{2(13)}{2} = 13$

ii. $T_1 = 5$ $T_2 = 8$ $T_3 = 11$

$S_3 = \frac{3(9+7)}{2} = 24$

$T_3 = 11$

iii. $T_n = a+(n-1)d$

$$= 5+(n-1)3$$
$$= 2+3n$$

b. $a = 4$ $S_\infty = 25r$

$$25r = \frac{4}{1-r}$$
$$25r - 25r^2 = 4$$
$$25r^2 - 25r + 4 = 0$$
$$(5r-1)(5r-4) = 0$$
$$r = \frac{1}{5}, \frac{4}{5}$$

c. i. $A = 10\,000 \times 1.1^n$

ii. $A_n = 10000 \times 1.1 + 10000 \times 1.1^2 + \ldots + 10000 \times 1.1^n$

$$= 10000\underbrace{(1.1+1.1^2+1.1^3+\ldots+1.1^n)}_{GP\ a=1.1\ r=1.1}$$
$$= 10000 \times \frac{1.1(1.1^n-1)}{1.1-1}$$
$$= \frac{10000(1.1)(1.1^n-1)}{0.1}$$

iii. $550\,000 = \frac{10000(1.1)(1.1^n-1)}{0.1}$

$$5 = 1.1^n - 1$$
$$6 = 1.1^n$$
$$\ln 6 = \ln 1.1^n$$
$$\ln 6 = n \ln 1.1$$
$$n = \frac{\ln 6}{\ln 1.1}$$
$$= 18.799$$

$\therefore$ She will make 19 deposits