

Student name:	••••••••••••••••••
	<u> </u>
747.4	Centre Number
	Student Number

2014
Higher School Certificate
Trial Examination

Mathematics

General Instructions:

- Reading time 5 minutes
- Working time 3 hours
- Write using black pen or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 16, show relevant mathematical reasoning and/or calculations
- Clearly identify the question you are answering on each writing booklet

Total marks - 100

Section I

10 Marks

- Attempt Questions 1 10
- Answer on the multiple choice answer sheet provided
- Allow about 15 minutes for this section

Section II

90 Marks

- Attempt Questions 11 16
- Allow about 2 hours and 45 minutes for this section

Total	/100
Question 16	/15
Question 15	/15
Question 14	/15
Question 13	/15
Question 12	/15
Question 11	/15
Multiple Choice	/10

Section I

10 marks

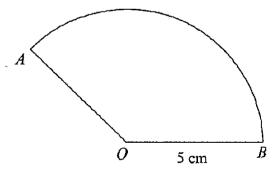
Attempt Questions 1 - 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

- 1 $4^2 \times 5^3 \times (5 \times 7^2)^4 \div 4^{-5}$ is equivalent to:
 - (A) $4^7 \times 5^7 \times 7^8$
 - (B) $4^{-3} \times 5^{7} \times 7^{6}$
 - (C) $4^7 \times 5^7 \times 7^6$
 - (D) 4⁻³ \times 5⁷ \times 7⁸
- 2 For the function $f(x) = \log_e x$, the value of $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ is:
 - (A) e^x
 - (B) $\log_e x$
 - (C) $\frac{1}{x}$
 - (D) $e \log_e x$
- 3 What is the size of each interior angle in a regular 20-sided polygon?
 - (A) 20°
 - (B) 162°
 - (C) 180°
 - (D) 3240°
- 4 Which of the following describes the locus of all points which are equidistant from A(2,3) and B(4,5)
 - (A) $(x-3)^2 + (y-4)^2 = 1$
 - (B) x + y 7 = 0
 - (C) $(x-3)^2 = 4(y-4)$
 - (D) x y + 1 = 0

- 5 What is the gradient of any line perpendicular to x + 3y = 2?
 - (A) -3
 - (B) $-\frac{1}{3}$
 - (C) $\frac{1}{3}$
 - (D) 3
- 6 For the function $y = \left| \frac{2}{x-3} \right|$, the Range is:
 - (A) $y \neq 2$
 - (B) $y \neq 0$
 - (C) y > 0
 - (D) $x \neq 3$
- 7 Which is **not** equal to cos 40°?
 - (A) cos 400°
 - (B) $\cos(-40^{\circ})$
 - (C) cos 220°
 - (D) cos 320°
- 8 If $f(x) = 4(7-2x)^3$ then f'(x) =
 - (A) $3(7-2x)^2$
 - (B) $-12(7-2x)^2$
 - (C) $21(7-2x)^2$
 - (D) $-24(7-2x)^2$
- 9 AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of 10π cm².



Not to scale

What is the arc length of the sector?

- (A) 2π cm
- (B) 4π cm
- (C) 6π cm
- (D) 10π cm

10 For the sequence $1, \frac{1}{x}, \frac{1}{x^2}, \dots$, the *n*th term is:

- (A) x^{1-n} (B) $\frac{1}{x^n}$ (C) $\frac{n}{x}$ (D) $\frac{1}{nx}$

END OF SECTION I

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 11 (15 marks)		Marks
(a) Evaluate tan 0.84 ^c correct to 3 signi	ficant figures.	1
(b) If $a + \sqrt{b} = 4(3 + 2\sqrt{5})$, find the val	ue of a and b .	2
(c) Find the exact value(s) of k for which x	$2^2 + 2kx + k + 2 = 0$ has real roots.	2
(d) The roots of the equation $2x^2 - x - 15 =$	0 are α and β . Find the value of:	
(i) $\alpha + \beta$		1
(ii) $\alpha\beta$		1
(iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$		2
(e) Differentiate $2x^3 \tan 4x$		2
(f) Evaluate $\int_{0}^{\frac{\pi}{3}} \sin 3x \ dx$		2
(g) Solve the equation $2 \cos x + 1 = 0$ in the	ne domain $0 \le x \le 2\pi$.	2

Solve the following equations:

(i)
$$|5x-2|=|2x+3|$$

2

(ii)
$$2^{x+1} = 8^x$$

(iv)

1

- For the parabola with equation $16y = x^2 4x 12$, find the:
 - (i) The coordinates of the vertex.

2

(ii) The coordinates of the focus.

1

(iii) The equation of the directrix. 1

The equation of the tangent to the parabola at the point $\left(0, \frac{-3}{4}\right)$

2

If the gradient of a function is given by $f'(x) = \frac{2x}{x^2-3}$, find f(x) given that f(2) = 4

2

- The fifth term of an arithmetic series is 42 and the twelfth term is 21.
 - (i)Find the common difference.

2

(iii) Find the sum of the first 80 terms.

1

(iii) Find the smallest value of n, such that $S_n < 0$

1

(a) Alex (A) and Bella (B) leave from point O at the same time. Alex travels at 20 km/h along a straight road in the direction 085° T. Bella travels at 25 km/h along another straight road in the direction 340° T.

Draw a diagram to represent this information.

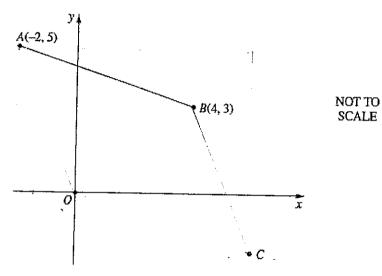
(i) Show that $\angle AOB$ is 105° where $\angle AOB$ is the angle between the directions taken by Alex and Bella.

1

(ii) Find the distance Alex and Bella are apart to the nearest kilometre after two hours.

2

(b)



The diagram shows the points A(-2,5), B(4,3) and O(0,0). The point C is the fourth vertex of the parallelogram OABC.

(i) Show that the equation of AB is x + 3y - 13 = 0

2

(ii) Find the exact perpendicular distance from O to the line AB.

1

(iii) Find the coordinates of C, such that OABC is a parallelogram.

1

(iv) Calculate the exact length of OC.

1

(v) Calculate the area of parallelogram OABC.

1

- (c) Let $f(x) = x^3 3x^2 9x + 22$
 - (i) Find the coordinates of the stationary points and determine their nature.

(ii) Find the coordinates of the point of inflexion.

1

2

- (iii) Sketch the graph of y = f(x) for $-3 \le x \le 6$, showing all relevant information,
- 2

(iv) Find the maximum value of f(x), for $-3 \le x \le 6$

1

(a) Evaluate:

$$\lim_{x \to \infty} \frac{x^4 + 3x^2 + 2}{5x^4 + 1}$$

1

(b) Evaluate the expression log_422 to 3 decimal places.

1

(c) Find $\frac{dy}{dx}$ if:

(i)
$$y = \frac{e^x}{x}$$

2

(ii)
$$y = x \log_e 2x$$

2

(d) Evaluate
$$\sum_{n=1}^{\infty} 3^{1-n}$$

2

(e) Simplify the expression
$$\frac{1}{2} log_e(x+3) - log_e \frac{x}{2}$$

2

(f) Sketch the function $y = log_e x$

1

(ii). Write down the domain of this function.

1

By using calculus, show that the function is always increasing.

1

(iv) Shade the region on your graph that satisfies:

2

$$y \ge log_e x$$
 and $(x-1)^2 + y^2 < 1$

Temp4U is an employment agency which specialises in contracting (a) temporary employees. They have analysed the number of job applications received over the last five years. They found that the demand (D), measured in hundreds, for temporary employment at time (t years) is given by the function:

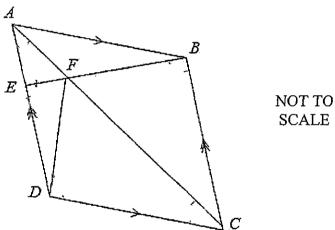
$$D(t) = 4\sin\left(\frac{\pi}{4}t\right) + 7$$

- State the amplitude and period of D(t) and sketch its graph for the (i) first twelve years.
- Using your graph, approximate the times in the first 12 years when (ii) the demand will be at its peak.

1

3

(b)



SCALE

ABCD is a rhombus, BE is perpendicular to AD and intersects AC at F.

Copy the diagram into your workbook.

(î). Explain why $\angle BCA = \angle DCA$.

1

(ii) Prove that $\triangle BFC$ and $\triangle DFC$ are congruent.

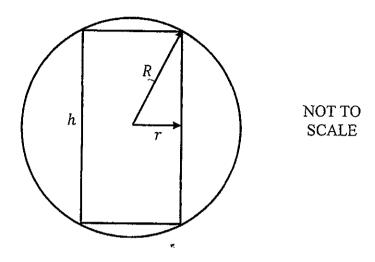
2

(jii; Show that $\angle FBC$ is a right angle.

- 1
- (iv). Hence or otherwise find the size of $\angle FDC$, giving reasons.
- 1

(d) A metal worker is required to cut a cylinder from a solid sphere of metal of radius R = 4 cm. The cylinder has a radius r cm and a height h cm.

The diagram below shows a cross section of the sphere and cylinder.



(i) Show that the volume $V \text{ cm}^3$, of the cylinder is given by

$$V = \frac{1}{4}\pi h(64 - h^2)$$

(ii) Hence, find the exact value of h, that gives the maximum volume of the cylinder.

- (a) Evaluate $\int_0^4 \sqrt{2 + x^4} dx$ using Simpson's rule with 3 function values. Answer to 2 decimal places.
- 2

- (b) An arithmetic sequence is defined by $S_n = 3n^2 11n$.
 - (i) Write down the expression for S_{n-1}

- 1
- (ii) Hence or otherwise write down the expression for the nth term, T_n
- 1

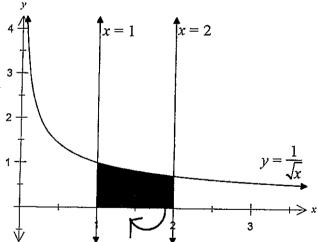
(c) (i) If $y = log_e(cos x)$, find $\frac{dy}{dx}$

1

(ii) Hence, find $\int \tan x \ dx$.

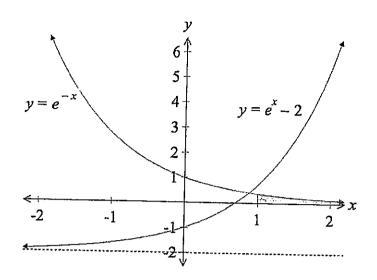
- 1
- (d) The region between the functions $y = \frac{1}{\sqrt{x}}$, x = 1 and x = 2 is rotated about the x-axis, as shown below.





Find the volume of the solid formed.

(e) The diagram below shows the graphs of $y = e^x - 2$ and $y = e^{-x}$



- (i) Find the area between the curves from x = 1 and x = 2. Leave your answer in terms of e.
- (ii) Show that the curves intersect when $e^{2x} 2e^x 1 = 0$.
- (iii) Find the x coordinate of the point of intersection of the curves.

 Answer correct to 3 decimal places.

 3

END OF EXAM

STANDARD INTEGRALS

$$\int x^{n} dx = \frac{1}{n+1} x^{n+1}, \ n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \ x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ \alpha \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x$, x > 0

1.A 2.C 3.B 4.B 5.D 6.C 7.C 8.D 9.B 10.A Question ! a) 1.12 b) a+ √6 = 12 + 8√5 = 12+ \320 a=12, b=320 c) $\Delta = b^2 - 4ac$ 70 for real roots (21)2-4x1x(k+2)70 4k2-4k-820 k2- k-2>0 (k-2)(k+1) >0 $k \gg 2$ and $k \ll -1$ $d) = -\frac{b}{a} = \frac{1}{2}$ 11) $\times B = \frac{C}{Q} = -\frac{15}{2} = -\frac{72}{2}$ 111) $\frac{1}{x^2} + \frac{1}{B^2} = \frac{B^2 + x^2}{x^2 B^2}$ $= \frac{(\beta + \alpha)^{2} - 2\alpha\beta}{(4\beta)^{2}}$ $= \frac{(\frac{1}{2})^{2} - 2x^{2}}{(\frac{1}{2})^{2}}$ (-7/2)= e) y' = Vdu + udv = 61' = 2x3. 4sec 4x+1an4x.6x $\frac{71/3}{1/3} = \frac{9x^3 \sec^2 4x + 6x^2 + an 4x}{5x^2 + an 4x}$ f $\int_{0}^{\pi/3} \sin 3x = \int_{0}^{\infty} \frac{-\cos 3x}{3} \int_{0}^{\pi/3}$ g) $\cos x = -\frac{1}{2}$ and $\frac{3}{1}$ and $\frac{3}{1}$ and $\frac{3}{1}$ cos T/3 = { x=受, 要 Question 12 a) 1) |5x-2 | = |2x+3| 5x-2=2x+3 or 5x-2=-(2x+3)x = 5 $x = 1²/₃ or <math>x = -\frac{1}{4}$ 3x = 5

a) 11) 2×+1=(23)x z+1 = 3x b) 16y=(x-2)2-16 (x-2)2=16(y+1) 1) vertex = (2,-1)n) focus= (2,3) n) directrix: y=-5 N) Y= 26 - 3 - 3 $y' = \frac{x}{3} - \frac{1}{4} = > M = -\frac{1}{4} \text{ at } x = 0$ $y - y_1 = M(x - x_1)$ $y + \frac{3}{4} = -\frac{1}{4}(x - 0)$ Y=-辛-辛 c) $f(x) = \log_{2}(x^{2}-3) + c$ 4 = loge(2-3)+c 4 = logel + C 4 = 0 + c => c=4 $f(x) = \log_{e}(x^{2}-3) + 4$ d)11a+4d=42 ... (1) a + 11d = 21 ... (2) (1) -(1) -7d = 21 d=-3, a=5411) $S_n = \frac{n}{2} \left[2a + (n-1)d \right]$ = 90 [2×54+ 79×3] = -5160 11) n [2a+(n-1)d] <0 $\frac{n}{2}$ [2x54+(-1):-3] <0 n [108-3n+3] <0 -3n2 + 111n <0 3n(-n+37) <0 n=0 (not valid) n=37 ... n=38 if sn <0 Question 13 a) (1) LAOB = 20°+:85° 0 20km/h A = 105°

Q13 cont $(a)^{n} |a|^{2} = b^{2} + c^{2} - 2bc \cos A$ a2 = 502 + 402 - 2x50 x 40 cos los =5063.8 $q=\sqrt{5063.8}=71$ km b) i) m of AB = 5-3 = equn: $y-y_1=m(x-x_1)$ $y-3=-\frac{1}{3}(x-4)$ 3y-q=-x+4x + 3y - 13 = 0. ii) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$ 4= b=3 $= \frac{0+0-13}{\sqrt{12+32}}$ x, =0 y,=0 $=\frac{13}{\sqrt{10}} = \frac{13\sqrt{10}}{10}$ on its 11) From AtoB translation of: down 2. From 0 - down 2 across 6 C = (6, -2) $d^2 = 6^2 + 2^2$ d = \10 = 2\100. v) A = lh $= 2\sqrt{10} \times \frac{13\sqrt{10}}{10}$ = 2602 c) $f(x) = x^3 - 3x^2 - 9x + 22$ $9f'(x) = 3x^2 - 6x - 9 = 0$ $3(x^2-2x-3)=0$ 3(x-3)(x+1)=0x=3,-1f''(x) = 6x - 6f''(3) = 12 > 0 min at (3,-5) f''(-1) = -12 < 0 max at (-1,27)11) inflexion pt when f 11/x =0 6x-6=0=>x=1

c) 11) cont test inflexion pt change in concavity ... inflexion pt = (1,11) (6,76)p m)_ (۱۱٫۱۱) $(v) f(6) = 6^3 - 3 \times 6^2 - 9 \times 6 + 22$. . max value = 76 Question 14 6) loge 22 = 2.23(loge 4 $\frac{dy = Vdv - vdv}{dx}$ $= x \cdot e^{x} - e^{x} \cdot 1$ $= \frac{x^{2} - e^{x}}{x^{2}}$ 1) dy = vdu + udv dx=x. = + loge2x,1 = 1+ loge 200 d) $T_1 = 3^{\circ} = 1$ $T_2 = 3^{-1} = \frac{1}{3}$ Q.P $T_3 = 3^{-2} = \frac{1}{3}$ Q = 1 $Q = \frac{1}{3}$ $5 \infty = \frac{q}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{3}{2}$ e) 1 loge (x+3) - loge = = loge (x+3)/2 $= \log_{e}\left(2\sqrt{x+3}\right)$ f) D 14 11) domain: (vi エフロ

Question 14 cont.

f) III) $y = log_{ex}$ Increasing => y' > 0 $y' = \frac{1}{x} > 0$ when x > 0(domain)

... $y = log_{ex}$ is always increasing

Question 15

i) $D(t) = 4\sin(\frac{\pi}{4}t) + 7$ amplitude = 4

period = $\frac{2\pi}{\pi/4}$ = 8

II 4D(t) $\pi/4$ II peak at t = 2, 10years

b) Given ABCD is a rhombus then diagonals bisect its angles : LBCA = LDCA ii) FC is common

LBCA = LDCA from i)

DC = BC (sides of rhombus)

... ABFC = ADFC (SAS)

II) LAEB = LEBC

(alternate L's AD | BC)

LAEB = 90° (given)

LEBC = 90° = night angle

IV) Since ABFC = ADFC

then LFDC = LEBC

(corresponding L's of congruent A's)

LEBC = 900 (from 111)

:. LFDC = 90°

 $(x) \int = \left[\frac{x^2}{2} + kx \right]^6 = 30$

18 +6k-0=30

k = 2

15 e)1) V= Tr2h (1)

By Pythagoras $r^2 + \left(\frac{h}{2}\right)^2 = R^2 \qquad R = 4$

Sub (2) inta (1)

 $V = \pi \left(\frac{16 - h^2}{4} \right) h$ $= \pi h \left(\frac{64 - h^2}{4} \right)$

= + Th (64-h2)

(3) $V = 16\pi h - \pi h^3$

 $\frac{dV}{dh} = \frac{16\pi - 3\pi h^{2}}{4} = 0$ $\frac{3h^{2}}{4} = \frac{16\pi}{4} = 0$ $h^{2} = \frac{64}{3}, h = \pm \frac{8}{\sqrt{3}}$

Since h70, h = 8/3 cm

Test for max $\frac{d^2V}{dh^2} = -\frac{6\pi h}{4}$

at h= 8/3, V" 40

.. Max Volume at h= 8/3 cm

Question 16

a) $\int = \frac{2-0}{6} \left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4) + 4f(3) + f(4) + 4\sqrt{83} + 2\sqrt{18} + 4\sqrt{83} + \sqrt{258} \right]$

= 23.11

 $b) 5_{n-1} = 3(n-1)^{2} - 11(n-1)$ $= 3(n^{2} - 2n+1) - 11n+11$ $= 3n^{2} - 17n + 14$

11) $T_n = S_n - S_{n-1}$ = $3n^2 - 11n - (3n^2 - 17n + 14)$ = 6n - 14

16.c) i)
$$y = \log_{e}(\cos x)$$

 $y' = -\sin x$ = $-\tan x$
 $\cos x$
11) $\int \tan x \, dx$
 $= -\log_{e}(\cos x) + c$
d) $V = \pi \int y^{2} \, dx$
 $= \pi \int \log_{e}(\cos x)$
 $= \pi \int \log_{e}(\cos x) + c$
d) $V = \pi \int y^{2} \, dx$
 $= \pi \int \log_{e}(\cos x) + c$
 $= \pi \int \log_{e}(\cos x) + c$
d) $V = \pi \int y^{2} \, dx$
 $= \pi \int \log_{e}(\cos x) + c$
 $= \pi \int$

 $e^{x} = 1+\sqrt{2}$ or $e^{x} = 1-\sqrt{2}$ $x = \log_{e}(1+\sqrt{2})$ or $x = \log_{e}(1-\sqrt{2})$ x = 0.1881no solution