

Name: _____

St George Girls High School

Trial Higher School Certificate Examination

2014



Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen.
- Begin each question in a new booklet
- Write your student number on each booklet.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11 – 16.
- Diagrams are not to scale.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 100

Section I – Pages 2 – 4

10 marks

- Attempt Questions 1 – 10 using the answer sheet provided at the end of the paper
- Allow about 15 minutes for this section

Section II – Pages 5 – 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks

Attempt Questions 1 to 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- The angle of inclination of the straight line $x + 3y - 5 = 0$ to the positive direction of the x -axis is closest to:
(A) 18°
(B) 162°
(C) 72°
(D) 108°
- The ratio of the lengths of the corresponding edges of two similar pyramids is 2:3. If the volume of the larger pyramid is 243 cm^3 , the volume of the smaller is:
(A) 81 cm^3
(B) 162 cm^3
(C) 108 cm^3
(D) 72 cm^3
- $y = f(x)$ is an odd function. The value of $\int_{-a}^a f(x) dx$ is:
(A) $f(a)$
(B) $2 \int_0^a f(x) dx$
(C) 0
(D) a
- The x -coordinates of the two stationary points on the curve $y = 2x^3 - 3x^2 - 12x + 18$ are:
(A) $x = -1, x = 2$
(B) $x = 1, x = -2$
(C) $x = 6, x = \frac{3}{2}$
(D) $x = -6, x = -\frac{3}{2}$

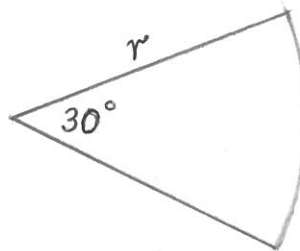
Section I (cont'd)

Marks

5. The area enclosed between the curves $y = 4 - x^2$ and $y = 4 - 2x$ is:

- (A) $\frac{16}{3}$ square units
- (B) $-\frac{16}{3}$ square units
- (C) $\frac{4}{3}$ square units
- (D) $\frac{2}{3}$ square units

6. The sector below has an area of $5\pi \text{ cm}^2$.



The value of r is:

- (A) $\sqrt{30} \pi \text{ cm}$
 - (B) $2\sqrt{15} \text{ cm}$
 - (C) $\sqrt{\frac{\pi}{6}} \text{ cm}$
 - (D) $\frac{1}{\sqrt{6}} \text{ cm}$
7. The equation $8^x = 32$ can be rewritten as:
- (A) $x = \log_8 5$
 - (B) $32 = \log_8 x$
 - (C) $8 = \log_x 32$
 - (D) $3x = \log_2 32$

Section I (cont'd)

Marks

8. The derivative of e^{x^3} is:
- (A) $x^2 e^{x^3} (3 + x)$
 - (B) e^{x^3}
 - (C) $3x^2 e^{x^3}$
 - (D) $\frac{e^{x^3}}{3x^2}$
9. A particle moves along a straight horizontal line with acceleration of $(2t - 1)$ m/s². Initially it is 3 metres to the right of the origin, moving with velocity of -2 m/s. The position of the particle after 3 seconds is:
- (A) 1.5 metres to the right of the origin
 - (B) 1.5 metres to the left of the origin
 - (C) 11.5 metres to the right of the origin
 - (D) 11.5 metres to the left of the origin
10. After an Electrical Engineering course at the UNSW, Matilda starts on a salary of \$65 000 with annual increments of 2.75%, so her consecutive salaries are:

$$\$65\,000, \quad \$65\,000 \times 1.0275, \quad \$65\,000 \times 1.0275^2, \quad \dots$$

What is total amount (to the nearest dollar) Matilda would earn during first 6 complete years of her employment?

- (A) \$390 041
- (B) \$417 816
- (C) \$419 010
- (D) \$479 238

End of Section I

Section II

90 Marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet	Marks
a) If $t = 0.23$, evaluate $\frac{1-t^2}{1+t^2}$, correct to three significant figures.	1
b) Evaluate $\ln 5$, correct to two decimal places.	1
c) Convert $\frac{5\pi}{7}$ to degrees and minutes.	1
d) Find $\int \left(\frac{1}{\sqrt{x}} - 3\right) dx$.	2
e) Solve $ 2x - 1 \leq 7$.	2
f) Differentiate $\frac{x^3}{e^x}$ and simplify the result fully.	2
g) Evaluate $\int_1^3 \frac{-x}{1+x^2} dx$.	3
h) Sketch the region defined by $(x + 3)^2 + (y - 3)^2 > 9$.	3

Question 12 (15 marks) Use a SEPARATE writing booklet

Marks

a) Differentiate

(i) $\tan 3x$.

1

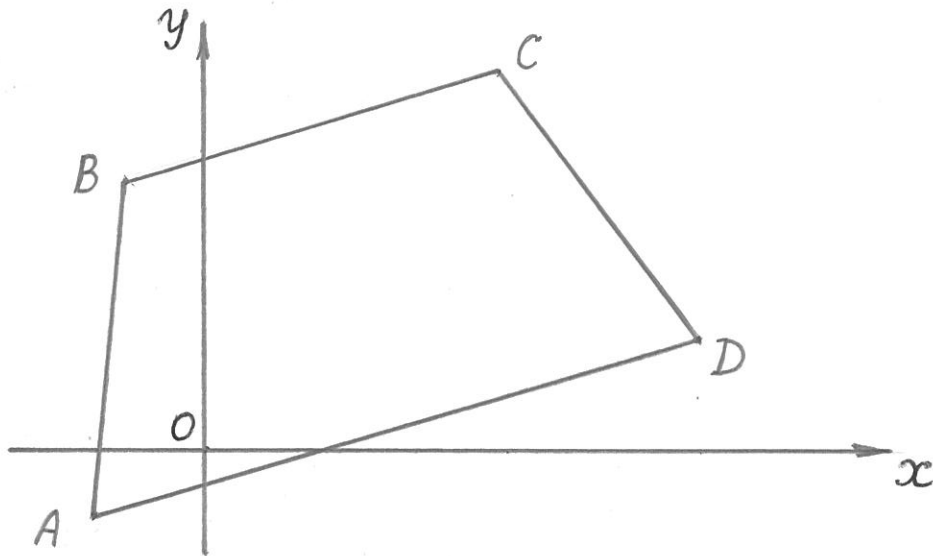
(ii) $\log\left(\frac{x^2+5}{x-3}\right)$.

2

(iii) $x e^{\sin x}$.

2

b) The points $A(-3, -3)$, $B(-2, 12)$, $C(8, 17)$ and $D(13, 5)$ form a trapezium, as shown on the diagram below (NOT TO SCALE).



(i) Show that the equation of the line through A and D is $x - 2y - 3 = 0$.

2

(ii) Show that the perpendicular distance from B to the line AD is $\frac{29}{\sqrt{5}}$ units.

2

(iii) Find the length of BC in exact form.

1

(iv) Show that $BC \parallel AD$.

1

(v) Point P (not shown) lies on the interval AD so that $AB \parallel PC$. Find the coordinates of the point P .

2

(vi) What type of a quadrilateral is $ABCP$?

1

(vii) Hence find the area of $ABCP$.

1

Question 13 (15 marks) Use a SEPARATE writing booklet

Marks

a) Consider the geometric series $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$

(i) Explain why the series has a limiting sum.

1

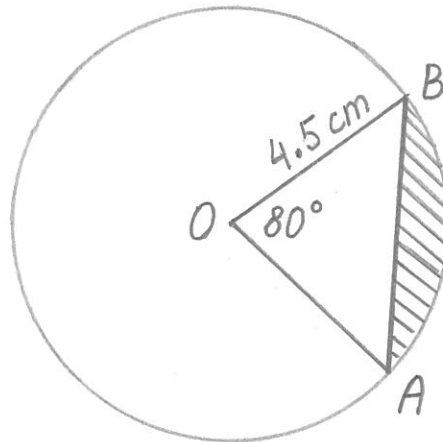
(ii) Find the limiting sum.

1

b) Find the equation of the tangent to the curve $y = \ln x$ at the point $(e^2, 2)$.

2

c)



In the diagram above O is the centre of the circle with radius 4.5 cm. The chord AB subtends an angle of 80° at the centre of the circle.

(i) Convert 80° into radians in terms of π .

1

(ii) Find the exact length of the minor arc AB .

1

(iii) Evaluate the area of the minor segment that has been shaded, correct to three decimal places.

2

Question 13 (cont'd)

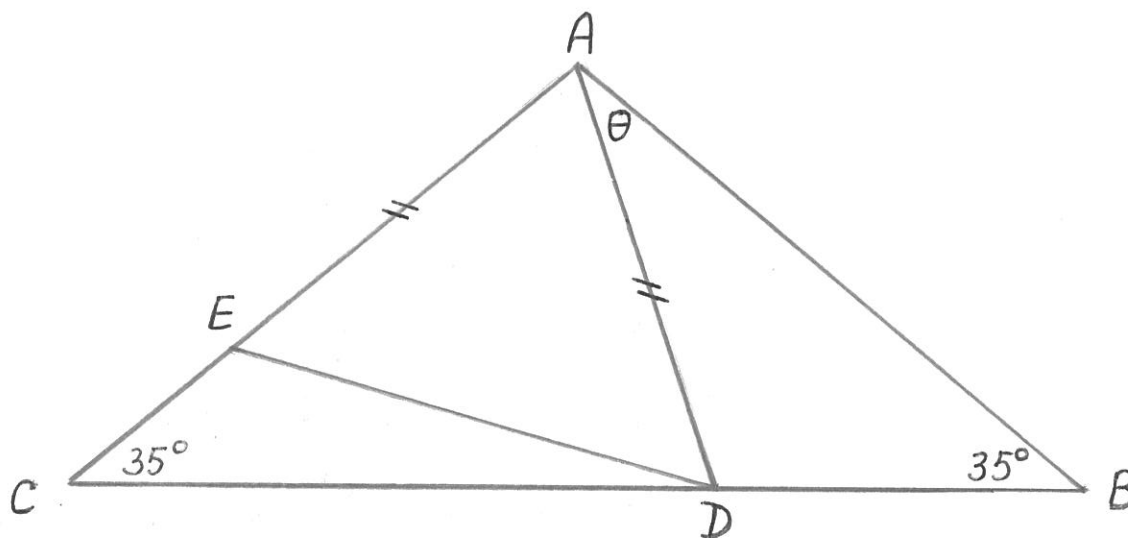
Marks

- d) The 7th term of an arithmetic sequence is 11 and the 21st term is 53.

2

Find the value of the common difference and the value of the first term of the sequence.

e)



In the diagram above $\triangle ABC$ is isosceles with $\angle ABC = \angle ACB = 35^\circ$, $AD = AE$ and $\angle DAB = \theta$.

- (i) Explain why $\angle ADC = 35^\circ + \theta$. 1

- (ii) Find the expression for $\angle CAD$ in terms of θ . 1

- (iii) Show that $\angle EDC = \frac{1}{2} \theta$. 3

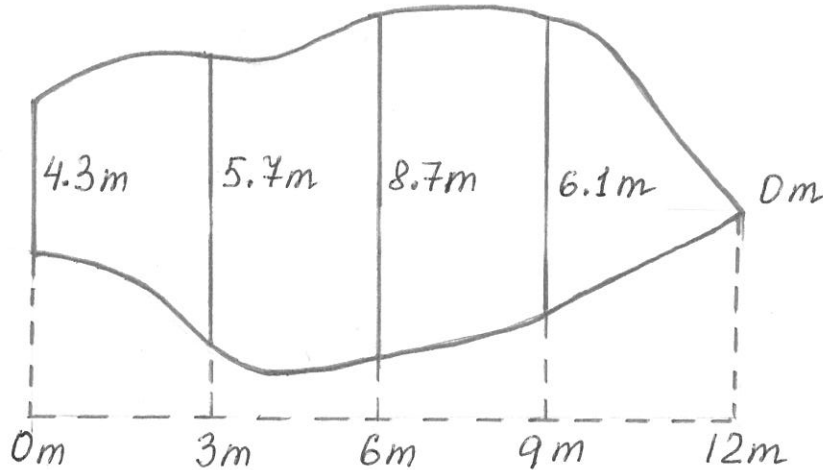
Question 14 (15 marks) Use a SEPARATE writing booklet

Marks

a) Solve $(\cos x - 3)(2 \cos x - 1) = 0$ in the domain $0 \leq x \leq 2\pi$.

3

b)



The diagram above shows the shape and dimensions of one of the ponds of a fishing farm.

3

Use the Simpson's Rule with all five given values to estimate the surface area of the pond to the nearest square.

c) A ball is thrown vertically up from the edge of a cliff 100 m above the valley floor. The initial velocity of the ball is 30 m/s. Consider the valley floor as the origin of space and the direction up as positive. The acceleration is always -10 m/s^2 .

(i) Show that the velocity is $v = 30 - 10t$.

1

(ii) Show that the displacement is $x = -5t^2 + 30t + 100$.

1

(iii) After how many seconds the ball is stationary?

1

(iv) How high is the ball above the valley floor at that instance?

1

(v) When is the ball at the cliff level again?

2

(vi) Show that the ball will hit the valley floor when $t = 3 + \sqrt{29}$ seconds.

2

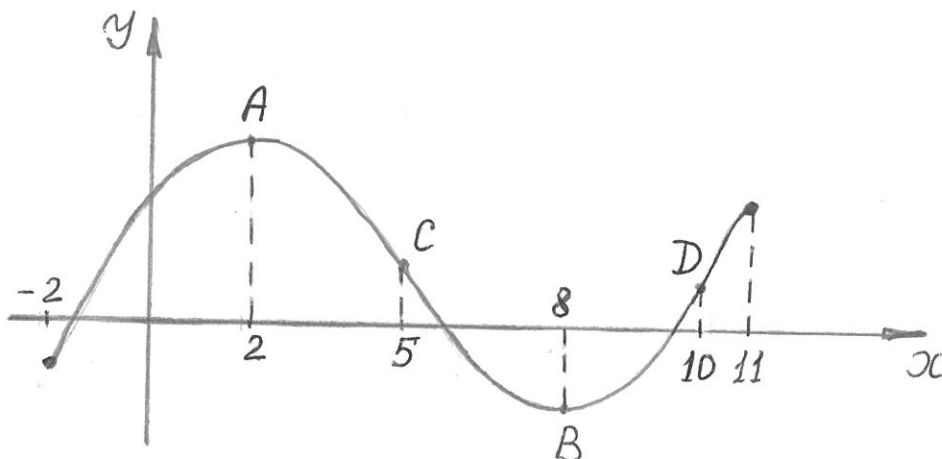
(vii) Find the speed of the impact, correct to the nearest metre per second.

1

Question 15 (15 marks) Use a SEPARATE writing booklet

Marks

a)



The diagram above shows the graph of a function $y = f(x)$ over the domain $-2 \leq x \leq 11$. A and B are its two stationary points and C and D are its two points of inflection.

State all the intervals of x for which:

(i) $f'(x) > 0$.

1

(ii) $f''(x) > 0$.

1

(iii) $f'(x) \times f''(x) > 0$.

2

b) A particle moves in a straight line such that at time t seconds its distance x metres from a fixed point O on the line is given by $x = 2 + \cos 3t$.

(i) What is the period of the motion?

1

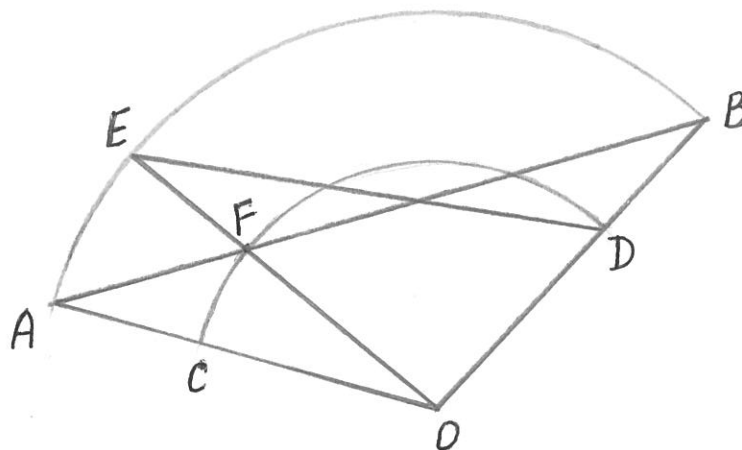
(ii) What is the furthest distance of the particle from point O ?

1

Question 15 (cont'd)

Marks

c)



In the diagram above O is the centre of two circular arcs AEB and CFD .

- (i) Copy the diagram into your booklet. 1
- (ii) Show that $\triangle OBF$ and $\triangle OED$ are congruent. 3
- (iii) Let $OC = a$ and $CA = b$. Show that $\triangle OCD$ and $\triangle OAB$ are similar and state the ratio of corresponding sides. 3
- (iv) The ratio of the area of $\triangle OCD$ to the area of $\triangle OAB$ is $49 : 144$. 2

Find the ratio of a to b .

Question 16 (15 marks) Use a SEPARATE writing booklet

Marks

- a) Find the stationary points on the curve $y = x^3(2 - x)$ and determine their nature. 4
- b) At the beginning of every month, starting on the 1st of January 2015, Grace plans to deposit \$1500 into a superannuation account, paying 6% interest per annum, compounded monthly.
- (i) Express the monthly interest rate as a decimal. 1
- (ii) Show that at the end of n months the total amount, A_n , on her superannuation account can be expressed as 3
- $$A_n = 301\,500(1.005^n - 1).$$
- (iii) If Grace continues with her superannuation scheme, what would be the total amount on her account on the 31st of December 2030? 2
- (iv) Grace has estimated that she would be able to retire after the total amount on her superannuation account reaches \$400 000. In this case, what would be the date of the first day of her retirement? 3
- (v) After a serious consideration Grace decided that she would retire on the 31st of December 2025 with the total of \$400 000, paying larger monthly instalments. Calculate her monthly instalments, correct to the nearest cent. 2

Student Name: _____

Class Teacher: _____

Section I

Year 12 Trial HSC Examination 2014

Mathematics

Multiple-choice Answer Sheet - Questions 1 - 10

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

A B C D
correct ↖

-
- | | | | | |
|-----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 2. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 3. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 4. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 5. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 6. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 7. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 8. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 9. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |
| 10. | A <input type="radio"/> | B <input type="radio"/> | C <input type="radio"/> | D <input type="radio"/> |

Trial HSC 2014 SOLUTIONS

Section 1

1. $y = -\frac{1}{3}x + 5$

$\tan \theta = -\frac{1}{3}$

$\theta = 162^\circ$

(B)

2. Volume ratio

$2^3 : 3^3$

$8 : 27$

$\frac{8}{27} = \frac{x}{243}$

$x = 72 \text{ cm}^3$

(D)

3. If $f(x)$ is odd.

$\int_{-a}^a f(x) dx = 0$

(C)

4. $y = 2x^3 - 3x^2 - 12x + 18$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$= 6(x^2 - x - 2)$

$= 6(x-2)(x+1)$

(A)

$\frac{dy}{dx} = 0$, when $x=2, x=-1$

5. To find x coordinates of the points of intersection.

$4 - x^2 = 4 - 2x$

$-x^2 + 2x = 0$

$-x(x-2) = 0$

$x=0, x=2$

Area = $\int_0^2 (4-x^2) - (4-2x) dx$

$= \int_0^2 4 - x^2 - 4 + 2x dx$

$= \left[-\frac{x^3}{3} + x^2 \right]_0^2$

$= -\frac{8}{3} + 4 = \frac{4}{3} \text{ u}^2$

(C)

6. $\frac{1}{2} \times \frac{\pi}{6} \times r^2 = 5\pi$

$r^2 = 12 \times 5$

$r = \sqrt{60}$

$r = 2\sqrt{15}$

(B)

7. $8^z = 32$

$2^{3z} = 2^5$

$\log_2 2^{3z} = \log_2 2^5$

$3z = \log_2 32$

(D)

8. $\frac{d}{dx} e^{x^3} = 3x^2 \cdot e^{x^3}$

(C)

9. $a = (2t-1) \text{ m/s}^2$

$v = \frac{2t^2}{2} - t + C$

when $t=0, v=-2$ (given)

$-2 = 0^2 - 0 - C$

$C = -2$

$\therefore v = t^2 - t - 2.$

$s = \frac{t^3}{3} - \frac{t^2}{2} - 2t + C$

when $t=0, s=3$ (given)

$\therefore s = \frac{t^3}{3} - \frac{t^2}{2} - 2t + 3$

when $t=3,$

$s = 9 - \frac{9}{2} - 6 + 3$

$= 6 - \frac{9}{2}$

$= 1.5 \text{ m to the right of origin.}$

(A)

10. $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{63000(1.0275^6 - 1)}{0.0275}$

$= \$417816$

(B)

Section II

Page 2

Question 11

$$(a) \frac{1 - 0.23^2}{1 + 0.23^2} = 0.89951\dots$$

$$= 0.900 \text{ (3 sig. fig)}$$

$$(b) \ln 5 = 1.61 \text{ (2 d.p)}$$

$$(c) \frac{5\pi}{7} = 128^\circ 34' 17.1''$$

$$= 128^\circ 34' \text{ (nearest minute)}$$

$$(d) \int \frac{1}{\sqrt{x}} - 3 \, dx$$

$$= \int x^{-1/2} - 3 \, dx$$

$$= \int x^{-1/2} - 3 \, dx$$

$$= \frac{x^{1/2}}{1/2} - 3x + C$$

$$= 2\sqrt{x} - 3x + C$$

$$(e) \begin{array}{l|l} 2x - 1 \leq 7 & -(x-1) \leq 7 \\ 2x \leq 8 & -2x + 1 \leq 7 \\ x \leq 4 & -2x \leq 6 \\ & x \geq -3 \end{array}$$

$$-3 \leq x \leq 4$$

$$(f) y = x^3 \cdot e^{-x}$$

$$y' = u'v + uv' \quad \left| \begin{array}{l} u = x^3 \\ u' = 3x^2 \\ v = e^{-x} \\ v' = -e^{-x} \end{array} \right.$$

$$= 3x^2 \cdot e^{-x} + x^3 \cdot (-e^{-x})$$

$$= 3x^2 e^{-x} - x^3 e^{-x}$$

$$= x^2 e^{-x} (3 - x)$$

Question 11

$$(g) \int_1^3 \frac{-x}{1+x^2} \, dx$$

$$= -\frac{1}{2} \int_1^3 \frac{2x}{1+x^2} \, dx$$

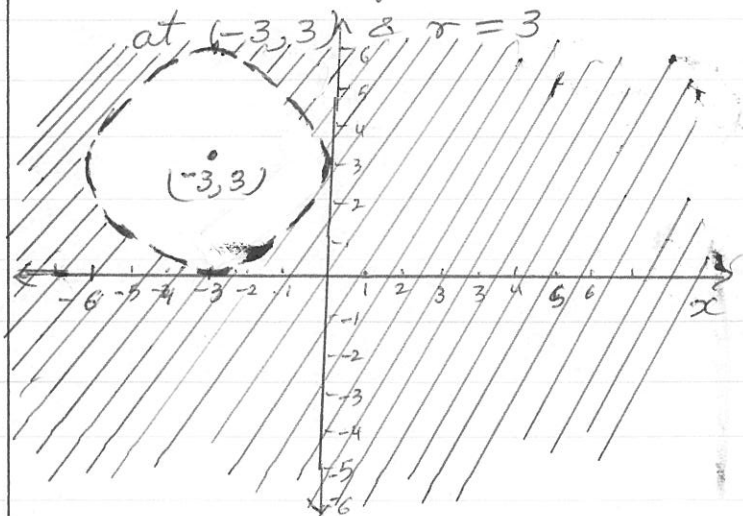
$$= -\frac{1}{2} [\log_e(1+x^2)]_1^3$$

$$= -\frac{1}{2} [\log_e 10 - \log_e 2]$$

$$= -\frac{1}{2} \log_e 5$$

$$(h) (x+3)^2 + (y-3)^2 > 9$$

the centre of the circle is at $(-3, 3)$ & $r = 3$



Question 12

Page 3

(a) (i) $y = \tan 3x$
 $\frac{dy}{dx} = 3 \sec^2(3x)$

(ii) $y = \log\left(\frac{x^2+5}{x-3}\right)$
 $y = \log(x^2+5) - \log(x-3)$
 $\frac{dy}{dx} = \frac{2x}{x^2+5} - \frac{1}{x-3}$

(iii) $y = x \cdot e^{\sin x}$

$y' = u'v + uv'$	$u = x$
$y' = e^{\sin x} + x \cos x e^{\sin x}$	$u' = 1$
	$v = e^{\sin x}$
	$v' = \cos x e^{\sin x}$

(b) (i) $(y-y_1) = m(x-x_1)$ | $m = \frac{8}{16}$
 $(y-5) = \frac{1}{2}(x-13)$ | $= \frac{1}{2}$
 $2y-10 = x-13$
 $x-2y-3 = 0$

(ii) for line $x-2y-3=0$ and point $B(-2, 12)$.

$$p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|1x - 2 + -2 \times 12 + -3|}{\sqrt{1+4}}$$

$$= \frac{29}{\sqrt{5}}$$

(iii) $BC^2 = (8-(-2))^2 + (17-12)^2$
 $= 100 + 25$
 $BC = \sqrt{125}$
 $= 5\sqrt{5}$ units.

(iv) gradient (BC) = $\frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{5}{10}$
 $= \frac{1}{2}$
 gradient (AD) = $\frac{1}{2}$ [from Part (i)]
 $m(BC) = m(AD) = \frac{1}{2}$
 $\therefore BC \parallel AD$

(v) B to A translation is 15 down and 1 left. Translating C 15 down and 1 left will give us the point P(7, 2)

(vi) $S(AB) = \sqrt{15^2 + 1^2}$
 $= \sqrt{226}$

$\therefore S(AB) \neq S(BC)$
 $\therefore ABCP$ is not a rhombus.
 hence $ABCP$ is a parallelogram.

(vii) Area $ABCP = \text{base} \times \text{height}$
 $= 5\sqrt{5} \times \frac{29}{\sqrt{5}}$
 $= 145 \text{ Unit}^2$

Question 13

(a) (i) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

$$r = \frac{\frac{2}{3}}{2} = \frac{\frac{2}{9}}{\frac{2}{3}}$$

$$\frac{2}{6} = \frac{2}{9} \times \frac{3}{2}$$

$$\frac{1}{3} = \frac{1}{3}$$

as $-1 < r (\frac{1}{3}) < 1$, the series will have a limiting sum.

(ii) $S_{\infty} = \frac{a}{1-r}$

$$= \frac{2}{1-\frac{1}{3}}$$

$$= \frac{2}{\frac{2}{3}}$$

$$= 3$$

(b) $y = \ln x$

$$\frac{dy}{dx} = \frac{1}{x}$$

equation of tangent.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{e^2}(x - e^2)$$

$$y - 2 = \frac{x}{e^2} - 1$$

$$y = \frac{x}{e^2} + 1 \quad \text{OR} \quad x - e^2y + e^2 = 0$$

(c) (i) $180^\circ = \pi$ radians

$$80^\circ = \frac{\pi}{180} \times 80$$

$$= \frac{4\pi}{9} \text{ radians.}$$

(ii) Arc length = $r\theta$
 $= 4.5 \times \frac{4\pi}{9}$
 $= 2\pi \text{ cm.}$

(iii) Area = $\frac{1}{2}r^2(\theta - \sin\theta)$
 $= \frac{1}{2} \times 4.5^2 \left(\frac{4\pi}{9} - \sin \frac{4\pi}{9} \right)$
 $\approx 10.125 (1.39626... - 0.984807)$
 $\approx 4.166 \text{ cm}^2 \text{ (3 d.p.)}$

(d) 7th term = 11

21st term = 53

$$d = \frac{53-11}{21-7}$$

$$= \frac{42}{14}$$

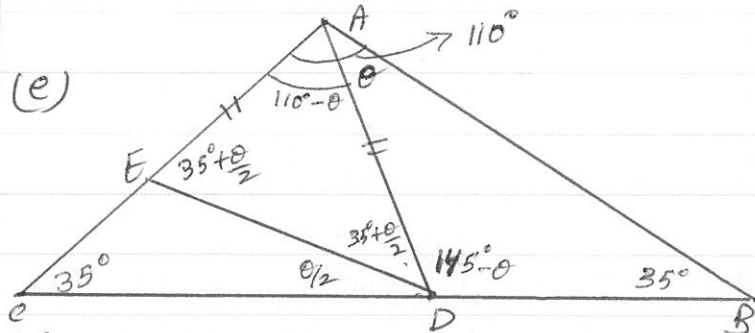
$$= 3.$$

$$T_n = a + (n-1)d$$

$$11 = a + (7-1) \times 3$$

$$11 = a + 18$$

$$a = -7$$



(i) $\angle ADC$ is exterior angle of $\triangle ADB$.

$$\therefore \angle ADC = \angle DAB + \angle DBA$$

(The exterior angle is equal to the sum of the two interior opposite angles.)

(ii) $\angle CAD = 180^\circ - 35^\circ - (\theta + 35^\circ)$

$$= 110^\circ - \theta$$

(iii) $\angle ADE = \frac{180^\circ - (110^\circ - \theta)}{2}$ (base angles of isosceles triangle)

$$= \frac{70^\circ + \theta}{2}$$

$$= 35^\circ + \frac{\theta}{2}$$

$$\angle ADB = 180^\circ - 35^\circ - \theta$$

$$= 145^\circ - \theta.$$

$$\therefore \angle EDC = 180^\circ - \left(\frac{70^\circ + \theta}{2} \right) - (145^\circ - \theta)$$

$$= 180^\circ - 35^\circ - \frac{\theta}{2} - 145^\circ + \theta$$

$$= \frac{\theta}{2}$$

OR

$$\hat{E}DC = \hat{A}DC - \hat{A}DE$$

$$= 35^\circ + \theta - \left(35^\circ - \frac{\theta}{2} \right)$$

$$= 35^\circ + \theta - 35^\circ + \frac{\theta}{2}$$

$$= \frac{\theta}{2}$$

Question 14

$$(a) (\cos x - 3)(2\cos x - 1) = 0$$

either $(\cos x - 3) = 0$ or $2\cos x - 1 = 0$

when $\cos x - 3 = 0$ when $2\cos x - 1 = 0$

$$\left. \begin{array}{l} \cos x = 3 \\ \text{no valid solution} \\ \text{as } \cos x \neq 3 \end{array} \right\} \begin{array}{l} \cos x = \frac{1}{2} \\ x = 60^\circ \text{ and} \\ x = 300^\circ \end{array}$$

$$(b) \text{Area} \doteq \frac{b-a}{6} [(f_1 + 4f_m + f_l) + (f_2 + 4f_m + f_l)]$$

$$\doteq \frac{6}{6} [(4.3 + 4 \times 5.7 + 8.7) + (8.7 + 4 \times 6.1 + 0)]$$

$$\doteq 68.9 \text{ m}^2$$

$$\doteq 69 \text{ m}^2$$

$$(c) a = -10 \text{ m/s}^2$$

$$(i) v = -10t + C$$

$$= -10t + 30$$

$$= 30 - 10t \quad \left(\begin{array}{l} \text{when } t=0, \\ v=30 \text{ m/s} \end{array} \right)$$

$$(ii) x = -\frac{10t^2}{2} + 30t + C$$

$$x = -5t^2 + 30t + 100 \quad \left(\begin{array}{l} \text{when } t=0, \\ x=100 \text{ m} \end{array} \right)$$

(iii) for the ball to be stationary

$$v = 0$$

$$-10t + 30 = 0$$

$$t = 3 \text{ seconds.}$$

(iv) when $t = 3$

$$x = -5 \times 9 + 30 \times 3 + 100$$

$$= -45 + 90 + 100$$

$$= 145 \text{ m above the valley floor.}$$

(v) when the ball is at cliff level again, $x = 100 \text{ m.}$

$$-5t^2 + 30t + 100 = 100$$

$$-5t^2 + 30t = 0$$

$$-t^2 + 6t = 0$$

$$(t) (-5t + 6) = 0$$

$$t = 0 \text{ and } t = 6$$

initial position | after 6 seconds the ball is again at the cliff level.

(vi) when the ball hits valley floor

$$x = 0$$

$$-5t^2 + 30t + 100 = 0$$

$$-t^2 + 6t + 20 = 0$$

$$-t^2 + 6t = -20$$

$$t^2 - 6t + 9 = 20 + 9$$

$$(t-3)^2 = 29$$

$$t-3 = \pm\sqrt{29}$$

$$t = 3 \pm \sqrt{29}$$

as $\sqrt{29} > 3$

$$3 - \sqrt{29} < 0,$$

$\therefore 3 - \sqrt{29}$ is not a valid value for t .

$$\therefore t = 3 + \sqrt{29} \text{ sec.}$$

(vii) when $t = 3 + \sqrt{29}$

$$v = -10(3 + \sqrt{29}) + 30$$

$$= -30 - 10\sqrt{29} + 30$$

$$= -10\sqrt{29}$$

$$\text{speed} = |v|$$

$$= |-10\sqrt{29}|$$

$$= 10\sqrt{29} \text{ m/s}$$

$$\doteq 54 \text{ m/s}$$

Question 15

(a) (i) $f'(x) > 0$ when
 $-2 < x < 2$ and
 $8 < x < 11$

(ii) $f''(x) > 0$ when
 $5 < x < 10$

(iii) $f'(x) \times f''(x) > 0$ when
 $2 < x < 5$ and when
 $8 < x < 10$

we find this by locating regions
 when $f'(x)$ & $f''(x)$ are either
 both positive or both negatives,
 as only only $-x- = +$ &
 $+x+ = +$.

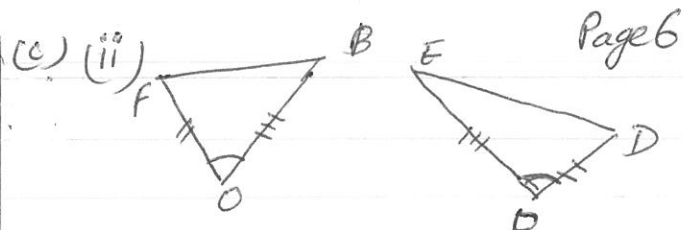
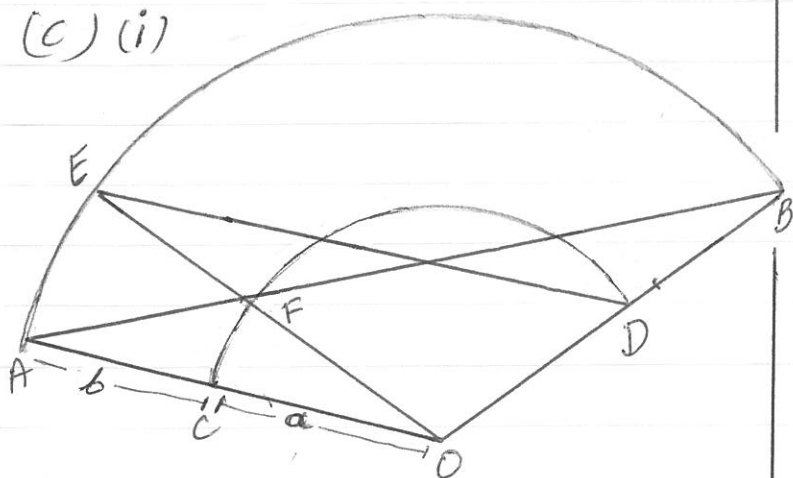
(b) $x = 2 + \cos 3t$

(i) $\frac{2\pi}{3}$

(ii) $2 + 1 = 3$ meters.

↳ This is the furthest point
 in positive direction for
 $\cos 3t$.

(c) (i)



in $\triangle OBF$ and $\triangle OED$:

$OF = OD$ (equal radii of arc OFD)

$OB = OE$ (equal radii of arc AEB)

$\angle FOB = \angle EOB$ (angle subtended at
 centre by arc EB)

$\therefore \triangle OBF \cong \triangle OED$ (SAS rule of
 congruency)

(iii) In $\triangle OCD$ and $\triangle OAB$,

$OC = OD = a$ (equal radii of
 arc CFD)

$OA = OB = a + b$ (equal radii of
 arc ACD)

$\therefore \frac{OC}{OA} = \frac{OD}{OB} = \frac{a}{a+b}$

$\angle COD = \angle AOB$ (common angle)

$\therefore \triangle OCD \sim \triangle OAB$ (two sides are in
 the same ratio
 and the included
 angles are equal -
 SAS rule of similarity)

(iv) let $a + b = c$

ratio of area = $a^2 : c^2$

\therefore ratio of side lengths = $\sqrt{a^2} : \sqrt{c^2}$

= $\sqrt{49} : \sqrt{144}$

= $7 : 12$

$a = 7$

$c = 12$

$b = 12 - 7 = 5$

ratio of a to b = $7 : 5$

Question 16

$$(a) \quad y = x^3(2-x) \quad \left| \begin{array}{l} u = x^3 \\ u' = 3x^2 \\ v = 2-x \\ v' = -1 \end{array} \right.$$

$$\frac{dy}{dx} = u'v + uv' = 3x^2(2-x) - x^3$$

$$= 6x^2 - 3x^3 - x^3$$

for stationary point

$$\frac{dy}{dx} = 0$$

$$\therefore 6x^2 - 4x^3 = 0$$

$$x^2(6-4x) = 0$$

\therefore stationary points are at

$$x = 0 \text{ \& } x = \frac{3}{2}$$

x	-1	0	1	$\frac{3}{2}$	2
y	-1	0	1	$\frac{27}{16}$	0
$\frac{dy}{dx}$	/	-	/	-	\

\therefore at $(0,0)$ we have a horizontal point of inflexion and at $(\frac{3}{2}, \frac{27}{16})$ we have a local maximum point.

$$(b) \text{ Interest rate} = \frac{6}{12 \times 100} = 0.005$$

Part
(ii) at the end of solution (last page)

(ii) 1st Jan 2015 to 31 Dec 2030 = 16 years

$$n = 16 \times 12 = 192$$

$$An = 301500(1.005^{192} - 1)$$

$$= \$484045.19$$

$$(iv) 301500(1.005^n - 1) = 400000$$

$$1.005^n = \frac{400000}{301500} + 1$$

$$\log 1.005^n = \log \left(\frac{701500}{301500} \right)$$

$$n = 169.312$$

$$\approx 170 \text{ months}$$

we can say just before 170 months or somewhere in the 170th months.

Since accounts are usually settled at the end of the month.

$$170 \div 12 = 14 \frac{2}{12} \text{ years}$$

or 14 years & 2 months.

\therefore 1st March of 2029 or sometime in February 2029.

$$(v) 1^{\text{st}} \text{ January 15 to } 31^{\text{st}} \text{ Dec 2025} = 11 \text{ years.}$$

$$400000 = \frac{M(1.005)(1.005^{132} - 1)}{0.005}$$

$$M(1.005)(1.005^{132} - 1) = 2000$$

$$M = \$2136.13$$

Let A_n be value of investment after n months

$$ii) A_1 = 1500 (1.005) \quad |$$

$$A_2 = A_1 (1.005) + 1500 (1.005) \\ = 1500 (1.005)^2 + 1500 (1.005)$$

$$A_3 = A_2 (1.005) + 1500 (1.005) \\ = 1500 (1.005)^3 + 1500 (1.005)^2 + 1500 (1.005)$$

$$A_n = 1500 (1.005) \left[1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1} \right] \\ = 1500 (1.005) \left[\frac{1.005^n - 1}{1.005 - 1} \right] \\ = 1507.50 \left[\frac{1.005^n - 1}{0.005} \right] \quad | \\ = 301500 \left[1.005^n - 1 \right]$$

$$A_3 = 1500$$